EIENSTEIN FIELD EQUATIONS AND HEISENBERG'S PRINCIPLE OF UNCERTAINLY THE CONSUMMATION OF GTR AND UNCERTAINTY PRINCIPLE

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ABSTRACT: The Einstein field equations (EFE) or Einstein's equations are a set of 10 equations in Albert Einstein's general theory of relativity which describe the fundamental interaction (e&eb) of gravitation as a result of space time being curved by matter and energy. First published by Einstein in 1915 as a tensor equation, the EFE equate spacetime curvature (expressed by the Einstein tensor) with (=) the energy and momentum tensor within that spacetime (expressed by the stress–energy tensor).Both space time curvature tensor and energy and momentum tensor is classified in to various groups based on which objects they are attributed to. It is to be noted that the total amount of energy and mass in the Universe is zero. But as is said in different context, it is like the Bank Credits and Debits, with the individual debits and Credits being conserved, holistically, the conservation and preservation of Debits and Credits occur, and manifest in the form of General Ledger. Transformations of energy also take place individually in the same form and if all such transformations are classified and written as a Transfer Scroll, it should tally with the total, universalistic transformation. This is a very important factor to be borne in mind. Like accounts are classifiable based on rate of interest, balance standing or the age, we can classify the factors and parameters in the Universe, be it age, interaction ability, mass, energy content. Even virtual particles could be classified based on the effects it produces. These aspects are of paramount importance in the study. When we write A+b+5, it means that we are adding A to B or B to A until we reach 5. Similarly, if we write A-B=0, it means we are taking away B from A and there may be time lag until we reach zero. There may also be cases in which instantaneous results are reached, which however do not affect the classification. By means of such a classification we obtain the values of Einstein Tensor and Momentum Energy Tensor, which are in fact the solutions to the Einstein's Field Equation. Terms "e" and "eb" are used for better comprehension of the lay reader. It has no other attribution or ascription whatsoever in the context of the paper. **For the sake of simplicity, we shall take the equality case of Heisenberg's Principle Of Uncertainty for easy consolidation and consubstantiation process. The "greater than" case can be attended to in a similar manner, with the symbolof"greater than" incorporated in the paper series**.

INTRODUCTION:

Similar to the way that electromagnetic fields are determined (eb) using charges and currents via Maxwell's equations, the EFE are used to determine the spacetime geometry resulting from the presence of mass-energy and linear momentum, that is, they (eb) determine the metric of spacetime for a given arrangement of stress–energy in the spacetime. The relationship between the metric tensor and the Einstein tensor allows the EFE to be written as a set of non-linear partial differential equations when used in this way. The solutions of the EFE are the components of the metric tensor. The inertial trajectories of particles and radiation (geodesics) in the resulting geometry are then calculated using the geodesic equation. As well as obeying local energy-momentum conservation, the EFE reduce to Newton's law of gravitation where the gravitational field is weak and velocities are much less than the speed of light. Solution techniques for the EFE include simplifying assumptions such as symmetry. Special classes of exact solutions are most often studied as they model many gravitational phenomena, such as rotating black holes and the expanding universe. Further simplification is achieved in approximating the actual spacetime as flat spacetime with a small deviation, leading to the linearised EFE. These equations are used to study phenomena such as gravitational waves.

Mathematical form

The Einstein field equations (EFE) may be written in the form:

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R the scalar curvature, $g_{\mu\nu}$ the metric tensor, Λ is the cosmological constant, G is Newton's gravitational constant, c the speed of light,in vacuum, and $T_{\mu\nu}$ the stress–energy tensor.

The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. Each tensor has 10 independent components. The four Bianchi identities reduce the number of independent equations from 10 to 6, leaving the metric with four gauge fixing degrees of freedom, which correspond to the freedom to choose a coordinate system.

Although the Einstein field equations were initially formulated in the context of a four-dimensional theory, some theorists have explored their consequences in n dimensions. The equations in contexts outside of general relativity are still referred to as the Einstein field equations. The vacuum field equations (obtained when T is identically zero) define Einstein manifolds. Despite the simple appearance of the equations they are, in fact, quite complicated. Given a specified distribution of(e&eb) matter and energy in the form of a stress–energy tensor, the EFE are understood to be equations for the metric tensor $g_{\mu\nu}$, as both the Ricci tensor and scalar curvature depend on the metric in a complicated nonlinear manner. In fact, when fully written out, the EFE are a system of 10 coupled, nonlinear, hyperbolic-elliptic partial differential equations.

One can write the EFE in a more compact form by defining the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu},$$

Which is a symmetric second-rank tensor that is a function of the metric? The EFE can then be written as

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

Using geometrized units where G = c = 1, this can be rewritten as

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi T_{\mu\nu}\ .$$

The expression on the left represents the curvature of spacetime as (eb) determined by the metric; the expression on the right represents the matter/energy content of spacetime. The EFE can then be interpreted as a set of equations dictating how matter/energy determines (eb) the curvature of spacetime.Or, curvature of space and time dictates the diffusion of matter energy. These equations, together with the geodesic equation, which dictates how freely-falling moves through space-time matter, form the core of the mathematical formulation of general relativity.

Sign convention

The above form of the EFE is the standard established by Misner, Thorne, and Wheeler. The authors analyzed all conventions that exist and classified according to the following three signs (S1, S2, and S3):

$$g_{\mu\nu} = [S1] \times \operatorname{diag}(-1,+1,+1,+1)$$

$${R^\mu}_{a\beta\gamma} = [S2] \times (\Gamma^\mu_{a\gamma,\beta} - \Gamma^\mu_{a\beta,\gamma} + \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\gamma a} - \Gamma^\mu_{\sigma\gamma}\Gamma^\sigma_{\beta a})$$

$$G_{\mu\nu} = [S3] \times \frac{8\pi G}{c^4} T_{\mu\nu}$$

The third sign above is related to the choice of convention for the Ricci tensor:

$$R_{\mu\nu} = [S2] \times [S3] \times {R^a}_{\mu a\nu}$$

With these definitions Misner, Thorne, and Wheeler classify themselves as $(+++)$, whereas Weinberg (1972) is $(+--)$, Peebles (1980) and Efstathiou (1990) are $(-++)$ while Peacock (1994), Rindler (1977), Atwater (1974), Collins Martin & Squires (1989) are $(-+-)$.

Authors including Einstein have used a different sign in their definition for the Ricci tensor which results in the sign of the constant on the right side being negative

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\frac{8\pi G}{c^4} T_{\mu\nu}.$$

The sign of the (very small) cosmological term would change in both these versions, if the +−−− metric sign convention is used rather than the MTW −+++ metric sign convention adopted here.

Equivalent formulations

Taking the trace of both sides of the EFE one gets

$$R - 2R + 4\Lambda = \frac{8\pi G}{c^4} T$$

Which simplifies to

$$-R + 4\Lambda = \frac{8\pi G}{c^4} T .$$

If one adds $-\frac{1}{2} g_{\mu\nu}$ times this to the EFE, one gets the following equivalent "trace-reversed" form

$$R_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} T\, g_{\mu\nu}\right) .$$

Reversing the trace again would restore the original EFE. The trace-reversed form may be more convenient in some cases (for example, when one is interested in weak-field limit and can replace $g_{\mu\nu}$ in the expression on the right with the Minkowski metric without significant loss of accuracy).

The cosmological constant

Einstein modified his original field equations to include a cosmological term proportional to the metric It is to be noted that even constants like gravitational field, cosmological constant, depend upon the objects for which they are taken in to consideration and total of these can be classified based on the parameterization of objects.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\, R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}\ .$$

The constant Λ is the cosmological constant. Since Λ is constant, the energy conservation law is unaffected.

The cosmological constant term was originally introduced by Einstein to allow for a static universe (i.e., one that is not expanding or contracting). This effort was unsuccessful for two reasons: the static universe described by this theory was unstable, and observations of distant galaxies by Hubble a decade later confirmed that our universe is, in fact, not static but expanding. So Λ was abandoned, with Einstein calling it the "biggest blunder [he] ever made". For many years the cosmological constant was almost universally considered to be 0.Despite Einstein's misguided motivation for introducing the cosmological constant term, there is nothing inconsistent with the presence of such a term in the equations. Indeed, recent improved astronomical techniques have found that a positive value of Λ is needed to explain the accelerating universe Einstein thought of the cosmological constant as an independent parameter, but its term in the field equation can also be moved algebraically to the other side, written as part of the stress–energy tensor:

$$T_{\mu\nu}^{(\mathrm{vac})} = -\frac{\Lambda c^4}{8\pi G}g_{\mu\nu}\ .$$

The resulting vacuum energy is constant and given by

$$\rho_{\mathrm{vac}} = \frac{\Lambda c^2}{8\pi G}$$

The existence of a cosmological constant is thus equivalent to the existence of a non-zero vacuum energy. The terms are now used interchangeably in general relativity.

Conservation of energy and momentum

General relativity is consistent with the local conservation of energy and momentum expressed as

$$\nabla_b T^{ab} \, = T^{ab}{}_{;b} \, = 0.$$

Derivation of local energy-momentum conservation

Which expresses the local conservation of stress–energy This conservation law is a physical requirement? With his field equations Einstein ensured that general relativity is consistent with this conservation condition.

Nonlinearity

The nonlinearity of the EFE distinguishes general relativity from many other fundamental physical theories. For example, Maxwell's equations of electromagnetism are linear in theelectric and magnetic fields, and charge and current distributions (i.e. the sum of two solutions is also a solution); another example is Schrödinger's equation of quantum mechanics which is linear in the wavefunction.

The correspondence principle

The EFE reduce to Newton's law of gravity by using both the weak-field approximation and the slow-motion approximation. In fact, the constant G appearing in the EFE is determined by making these two approximations.

Vacuum field equation

If the energy-momentum tensor $T_{\mu\nu}$ is zero in the region under consideration, then the field equations are also referred to as the vacuum field equations. By setting $T_{\mu\nu} = 0$ in the trace - reversed field equations, the vacuum equations can be written as

$$R_{\mu\nu} = 0 \,.$$

In the case of nonzero cosmological constant, the equations are

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \,.$$

The solutions to the vacuum field equations are called vacuum solutions. Flat Minkowski space is the simplest example of a vacuum solution. Nontrivial examples include the Schwarzschild solution and the Kerr solution.

Manifolds with a vanishing Ricci tensor, $R_{\mu\nu} = 0$, are referred to as Ricci-flat manifolds and manifolds with a Ricci tensor proportional to the metric as Einstein manifolds.

Einstein–Maxwell equations

If the energy-momentum tensor $T_{\mu\nu}$ is that of an electromagnetic field in free space, i.e. if the electromagnetic stress–energy tensor

$$T^{\alpha\beta} = \, -\frac{1}{\mu_0} \left(F^{\alpha\psi} F_{\psi}{}^{\beta} + \frac{1}{4} g^{\alpha\beta} F_{\psi\tau} F^{\psi\tau} \right)$$

is used, then the Einstein field equations are called the Einstein–Maxwell equations (with cosmological constant Λ, taken to be zero in conventional relativity theory):

$$R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} + g^{\alpha\beta} \Lambda = \frac{8\pi G}{c^4 \mu_0} \left(F^{\alpha\psi} F_{\psi}{}^{\beta} + \frac{1}{4} g^{\alpha\beta} F_{\psi\tau} F^{\psi\tau} \right).$$

Additionally, the covariant Maxwell Equations are also applicable in free space:

$$F^{\alpha\beta}{}_{;\beta} = 0$$

$$F_{[\alpha\beta;\gamma]} = \frac{1}{3} \left(F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} \right) = 0.$$

Where the semicolon represents a covariant derivative, and the brackets denote anti-summarization. The first equation asserts that the 4-divergence of the two-form F is zero, and the second that its exterior derivative is zero. From the latter, it follows by the Poincaré lemma that in a coordinate chart it is possible to introduce an electromagnetic field potential such that

$$F_{\alpha\beta} = A_{\alpha;\beta} - A_{\beta;\alpha} = A_{\alpha,\beta} - A_{\beta,\alpha}$$

In which the comma denotes a partial derivative. This is often taken as equivalent to the covariant Maxwell equation from which it is derived however, there are global solutions of the equation which may lack a globally defined potential.[9]

Solutions

The solutions of the Einstein field equations are metrics of spacetime. The solutions are hence often called 'metrics'. These metrics describe the structure of the spacetime including the inertial motion of objects in the spacetime. As the field equations are non-linear, they cannot always be completely solved (i.e. without making approximations). For example, there is no known complete solution for a spacetime with two massive bodies in it (which is a theoretical model of a binary star system, for example). However, approximations are usually made in these cases. These are commonly referred to as post-Newtonian approximations. Even so, there are numerous cases where the field equations have been solved completely, and those are called exact solutions. The study of exact solutions of Einstein's field equations is one of the activities of cosmology. It leads to the prediction of black holes and to different models of evolution of the universe.

The linearised EFE

The nonlinearity of the EFE makes finding exact solutions difficult. One way of solving the field equations is to make an approximation, namely, that far from the source(s) of gravitating matter, the gravitational field is very weak and the spacetime approximates that of Minkowski space. The metric is then written as the sum of the Minkowski metric and a term representing the deviation of the true metric from the Minkowski metric. This linearization procedure can be used to discuss the phenomena of gravitational radiation.

Ricci curvature

In differential geometry, the Ricci curvature tensor, named after Gregorio Ricci-Curbastro, represents the amount by which the volume element of a geodesic ball in a curved Riemannian deviates from that of the standard ball in Euclidean space. As such, it provides one way of measuring the degree to which the geometry determined by a given Riemannian metric might differ from that of ordinary Euclidean n-space. The Ricci tensor is defined on any pseudo-Riemannian manifold, as a trace of the Riemann curvature tensor. Like the metric itself, the Ricci tensor is a symmetric bilinear form on the tangent space of the manifold (Besse 1987, p. 43).

In relativity theory, the Ricci tensor is the part of the curvature of space-time that determines the degree to which matter will tend to converge or diverge in time (via the Raychaudhuri equation). It is related to the matter content of the universe by means of the Einstein field equation. In differential geometry, lower bounds on the Ricci tensor on a Riemannian manifold allow one to extract global geometric and topological information by comparison (cf. comparison theorem) with the geometry of a constant curvature space form. If the Ricci tensor satisfies the vacuum Einstein equation, then the manifold is an Einstein manifold, which has been extensively studied (cf. Besse 1987). In this connection, the flow equation governs the evolution of a given metric to an Einstein metric, the precise manner in which this occurs ultimately leads to the solution of the Poincaré conjecture.

Scalar curvature

In Riemannian geometry, the scalar curvature (or Ricci scalar) is the simplest curvature invariant of a Riemannian manifold. To each point on a Riemannian manifold, it assigns a single real number determined by the intrinsic geometry of the manifold near that point. Specifically, the scalar curvature represents the amount by which the volume of a geodesic ball in a curved Riemannian manifold deviates from that of the standard ball in Euclidean space. In two dimensions, the scalar curvature is twice the Gaussian curvature, and completely characterizes the curvature of a surface. In more than two dimensions, however, the curvature of Riemannian manifolds involves more than one functionally independent quantity.

In general relativity, the scalar curvature is the Lagrangian density for the Einstein–Hilbert action.

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The Euler–Lagrange equations for this Lagrangian under variations in the metric constitute the vacuum Einstein field equations, and the stationary metrics are known as Einstein metrics. The scalar curvature is defined as the trace of the Ricci tensor, and it can be characterized as a multiple of the average of the sectional curvatures at a point. Unlike the Ricci tensor and sectional curvature, however, global results involving only the scalar curvature are extremely subtle and difficult. One of the few is the positive mass theorem of Richard Schoen, Shing-Tung Yau and Edward Witten. Another is the Yamabe problem, which seeks extremal metrics in a given conformal class for which the scalar curvature is constant.

Metric tensor (general relativity)

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$$\begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix}$$

Metric tensor of spacetime in general relativity written as a matrix.

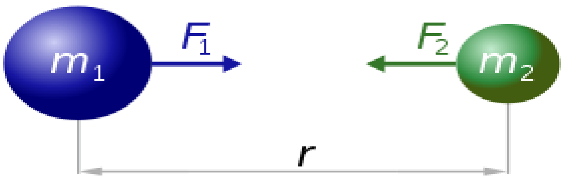
In general relativity, the metric tensor (or simply, the metric) is the fundamental object of study. It may loosely be thought of as a generalization of the gravitational field familiar from Newtonian gravitation. The metric captures all the geometric and causal structure inspacetime, being used to define notions such as distance, volume, curvature, angle, future and past.

Notation and conventions: Throughout this article we work with a metric signature that is mostly positive (− + + +); see sign convention. As is customary in relativity, units are used where the speed of light c = 1. The gravitation constant G will be kept explicit. The summation, where repeated indices are automatically summed over, is employed.

COSMOLOGICAL CONSTANT:

In physical cosmology, the cosmological constant (usually denoted by the Greek capital letter lambda: Λ) was proposed by Albert Einstein as a modification of his original theory of general relativity to achieve a stationary universe. Einstein abandoned the concept after the observation of the Hubble redshift indicated that the universe might not be stationary, as he had based his theory on the idea that the universe is unchanging. However, the discovery of cosmic acceleration in 1998 has renewed interest in a cosmological constant.

Gravitational constant



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

The gravitational constant denoted by letter G, is an empirical physical constant involved in the calculation(s) of gravitational force between two bodies.G should not be confused with "little g" (g), which is the local gravitational field (equivalent to the free-fall acceleration, especially that at the Earth's surface..

Speed of light.

The speed of light in vacuum, usually denoted by c, is a universal physical constant important in many areas of physics. Its value is 299,792,458 metres per second, a figure that is exact since the length of the metre is defined from this constant and the international standard for time. In imperial units this speed is approximately 186,282 miles per second. According to special relativity, c is the maximum speed at which all energy, matter, and information in the universe can travel. It is the speed at which all massless particles and associated fields (including electromagnetic radiation such aslight) travel in vacuum. It is also the speed of gravity (i.e. of gravitational waves) predicted by current theories. Such particles and waves travel at c regardless of the motion of the source or the inertial frame of reference of the observer. In the Theory, c interrelates space and time, and also appears in the famous equation of mass–energy equivalence = mc2

The speed at which light propagates through transparent materials, such as glass or air, is less than c. The ratio between cand the speed v at which light travels in a material is called the refractive index n of the material (n = c / v). For example, for visible light the refractive index of glass is typically around 1.5, meaning that light in glass travels atc / 1.5 ≈ 200,000 km/s; the refractive index of air for visible light is about 1.0003, so the speed of light in air is about90 km/s slower than c.

In most practical cases, light can be thought of as moving "instantaneously", but for long distances and very sensitive measurements the finite speed of light has noticeable effects. In communicating with distant space probes, it can take minutes to hours for a message to get from Earth to the spacecraft or vice versa. The light we see from stars left them many years ago, allowing us to study the history of the universe by looking at distant objects. The finite speed of light also limits the theoretical maximum speed of computers, since information must be sent within the computer from chip to chip. Finally, the speed of light can be used with time of flight measurements to measure large distances to high precision.

Ole Rømer first demonstrated in 1676 that light travelled at a finite speed (as opposed to instantaneously) by studying the apparent motion of Jupiter's moon Io. In 1865, James Clerk Maxwell proposed that light was an electromagnetic wave, and therefore travelled at the speed c appearing in his theory of electromagnetism In 1905, Albert Einstein postulated that the speed of light with respect to any inertial frame is independent of the motion of the light source and explored the consequences of that postulate by deriving the special theory of relativity and showing that the parameter c had relevance outside of the context of light and electromagnetism. After centuries of increasingly precise measurements, in 1975 the speed of light was known to be 299,792,458 m/s with a measurement uncertainty of 4 parts per billion. In 1983, the metrewas redefined in the International System of Units (SI) as the distance travelled by light in vacuum in 1/299,792,458 of asecond. As a result, the numerical value of c in metres per second is now fixed exactly by the definition of the metre

] Numerical value, notation, and units

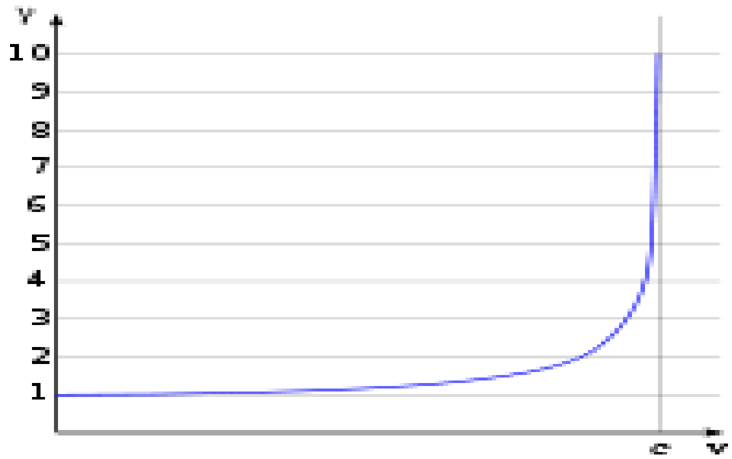
The speed of light in vacuum is usually denoted by c, for "constant" or the Latin celeritas (meaning "swiftness"). Originally, the symbol V was used, introduced by James Clerk Maxwell in 1865. In 1856, Wilhelm Eduard Weber and Rudolf Kohlrausch used c for a constant later shown to equal √2 times the speed of light in vacuum. In 1894, Paul Druderedefined c with its modern meaning. Einstein used V in his original German-language papers on special relativity in 1905, but in 1907 he switched to c, which by then had become the standard symbol.

Sometimes c is used for the speed of waves in any material medium, and c0 for the speed of light in vacuum This subscripted notation, which is endorsed in official SI literature, has the same form as other related constants: namely, μ0for the vacuum permeability or magnetic constant, ε0 for the vacuum permittivity or electric constant, and Z0 for the impedance. This article uses c exclusively for the speed of light in vacuum.

In the International System of Units (SI), the metre is defined as the distance light travels in vacuum in 1/299,792,458 of a second. This definition fixes the speed of light in vacuum at exactly 299,792,458 m/s. As a dimensional physical constant, the numerical value of c is different for different unit systems. In branches of physics in which cappears often, such as in relativity, it is common to use systems of natural units of measurement in which c = 1 Using these units, c does not appear explicitly because multiplication or division by 1 does not affect the result.

Fundamental role in physics

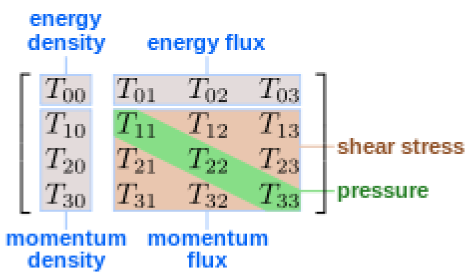
Speed at which light waves propagate in vacuum is independent both of the motion of the wave source and of the inertial frame of reference of the observer This invariance of the speed of light was postulated by Einstein in 1905 after being motivated by Maxwell's theory of electromagnetism and the lack of evidence for the luminiferous aether; it has since been consistently confirmed by many experiments. It is only possible to verify experimentally that the two-way speed of light (for example, from a source to a mirror and back again) is frame-independent, because it is impossible to measure the one-way speed of light (for example, from a source to a distant detector) without some convention as to how clocks at the source and at the detector should be synchronized. However, by adopting Einstein synchronization for the clocks, the one-way speed of light becomes equal to the two-way speed of light by definition. The special theory of relativity explores the consequences of this invariance of c with the assumption that the laws of physics are the same in all inertial frames of reference. One consequence is that c is the speed at which all massless particles and waves, including light, must travel in vacuum.



The Lorentz factor γ as a function of velocity. It starts at 1 and approaches infinity as v approaches c.

Special relativity has many counterintuitive and experimentally verified implications These include the equivalence of mass and energy (E = mc2), length contraction (moving objects shorten),[and time dilation (moving clocks run slower). The factor γ by which lengths contract and times dilate, is known as the Lorentz factor and is given by γ = (1 − v2/c2)−1/2, where v is the speed of the object. The difference of γ from 1 is negligible for speeds much slower than c, such as most everyday speeds—in which case special relativity is closely approximated by Galilean relativity—but it increases at relativistic speeds and diverges to infinity as v approaches c.The results of special relativity can be summarized by treating space and time as a unified structure known as spacetime (with c relating the units of space and time), and requiring that physical theories satisfy a special symmetry called Lorentz invariance, whose mathematical formulation contains the parameter c Lorentz invariance is an almost universal assumption for modern physical theories, such as quantum electrodynamics, quantum chromodynamics, the Standard Model of particle physics, and general relativity. As such, the parameter c is ubiquitous in modern physics, appearing in many contexts that are unrelated to light. For example, general relativity predicts that c is also the speed and of gravitational waves In non-inertial frames of reference (gravitationally curved space or accelerated reference frames), the local speed of light is constant and equal to c, but the speed of light along a trajectory of finite length can differ from c, depending on how distances and times are defined. It is generally assumed that fundamental constants such as c have the same value throughout spacetime, meaning that they do not depend on location and do not vary with time. However, it has been suggested in various theories that the speed of light may have changed over time No conclusive evidence for such changes has been found, but they remain the subject of ongoing research. It also is generally assumed that the speed of light is isotropic, meaning that it has the same value regardless of the direction in which it is measured. Observations of the emissions from nuclear energy levels as a function of the orientation of the emitting nuclei in a magnetic field (see Hughes–Drever experiment), and of rotating optical resonators (see Resonator experiments) have put stringent limits on the possible two-way anisotropy.

Stress–energy tensor



The components of the stress-energy tensor.

The stress–energy tensor (sometimes stress–energy–momentum tensor) is a tensor quantity in physics that describes the density and flux of energy and momentum in spacetime, generalizing the stress tensor of Newtonian physics. It is an attribute of matter, radiation, and non-gravitational force fields. The stress-energy tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass is the source of such a field in Newtonian gravity.

The stress–energy tensor involves the use of superscripted variables which are not exponents (see Einstein summation notation). The components of the position four-vector are given by: x0 = t (time in seconds), x1 = x (in meters), x2 = y (in meters), and x3 = z(in meters).

The stress–energy tensor is defined as the tensor $T^{\alpha\beta}$ of rank two that gives the flux of the αth component of the momentum vector across a surface with constant xβ coordinate. In the theory of relativity, this momentum vector is taken as the four-momentum. In general relativity, the stress-energy tensor is symmetric

$$T^{\alpha\beta} = T^{\beta\alpha}.$$

In some alternative theories like Einstein–Cartan theory, the stress–energy tensor may not be perfectly symmetric because of a nonzero spin tensor, which geometrically corresponds to a nonzero torsion tensor.

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Identifying the components of the tensor

In the following i and k range from 1 through 3.

The time–time component is the density of relativistic mass, i.e. the energy density divided by the speed of light squared It is of special interest because it has a simple physical interpretation. In the case of a perfect fluid this component is

$$T^{00} = \rho ,$$

And for an electromagnetic field in otherwise empty space this component is

$$T^{00} = \frac{\epsilon_0}{2}(\frac{E^2}{c^2} + B^2)$$

Where E and B are the electric and magnetic fields respectively

The flux of relativistic mass across the xi surface is equivalent to the density of the ith component of linear momentum,

$$T^{0i} = T^{i0} .$$

The components

$$T^{ik}$$

Represent flux of ith component of linear momentum across the xk surface. In particular,

$$T^{ii}$$

(Not summed) represents normal stress which is called pressure when it is independent of direction. Whereas

$$T^{ik} , \quad i \neq k$$

Represents shear stress (compare with the stress tensor).

In solid state physics and fluid mechanics, the stress tensor is defined to be the spatial components of the stress–energy tensor in the comoving frame of reference. In other words, the stress energy tensor in engineering differs from the stress energy tensor here by a momentum convective term.

GOVERNING EQUATIONS

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

FIRST TERM

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13} \qquad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14} \qquad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15} \qquad 3$$

SECOND TERM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - (b'_{13})^{(1)} T_{13} \qquad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - (b'_{14})^{(1)} T_{14} \qquad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - (b'_{15})^{(1)} T_{15} \qquad 6$$

THIRD TERM

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - (a'_{16})^{(2)} G_{16} \qquad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - (a'_{17})^{(2)} G_{17} \qquad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - (a'_{18})^{(2)} G_{18} \qquad 9$$

FOURTH TERM

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - (b'_{16})^{(2)} T_{16} \qquad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - (b'_{17})^{(2)} T_{17} \qquad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - (b'_{18})^{(2)} T_{18} \qquad 12$$

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS HOLISTIC SYSTEM: EINSTEIN FIELD EQUATION WITH ALL THE FOUR TERMS:

FIRST TERM

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \right] G_{13} \qquad 13$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \right] G_{14} \qquad 14$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \right] G_{15} \qquad 15$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

SECOND TERM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \right] T_{13} \qquad 16$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \right] T_{14} \qquad 17$$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \right] T_{15}$ 18

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

THIRD TERM AND FOURTH TERM

THIRD TERM

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \right] G_{16}$ 19

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \right] G_{17}$ 20

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \right] G_{18}$ 21

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

FOURTH TERM

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \right] T_{16}$ 22

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \right] T_{17}$ 23

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \right] T_{18}$ 24

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detritions coefficients for category 1, 2 and 3

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

THRID TERM

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{-(a''_{13})^{(1,1)}(T_{14},t)} \right] G_{16}$ 25

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{-(a''_{14})^{(1,1)}(T_{14},t)} \right] G_{17}$

26

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{-(a''_{15})^{(1,1)}(T_{14},t)} \right] G_{18}$ 27

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{-(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{15})^{(1,1)}(T_{14},t)}$ are second detritions coefficients for category 1, 2 and 3

FIRST TERM

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}\right]T_{13}$ 28

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}\right]T_{14}$ 29

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}\right]T_{15}$ 30

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}$ are second augmentation coefficients for category 1, 2 and 3

FIRST TERM

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\right]G_{13}$ 31

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\right]G_{14}$ 32

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\right]G_{15}$ 33

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

FOURTH TERM

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\right]T_{16}$ 34

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\right]T_{17}$ 35

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\right]T_{18}$ 36

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

FOURTH TERM

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1)}(G,t)}\right]T_{16}$ 37

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1)}(G,t)}\right]T_{17}$ 38

$\frac{dT_{6}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1)}(G,t)}\right]T_{18}$ 39

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detritions coefficients for category 1, 2 and 3

FIRST TERM

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2)}(T_{17},t)} \right] G_{13}$$ 40

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2)}(T_{17},t)} \right] G_{14}$$ 41

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2)}(T_{17},t)} \right] G_{15}$$ 42

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

THIRD TERM

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \right] G_{16}$$ 43

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \right] G_{17}$$ 44

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \right] G_{18}$$ 45

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

SECOND TERM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \right] T_{13}$$ 46

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \right] T_{14}$$ 47

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \right] T_{15}$$ 48

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detritions coefficients for category 1, 2 and 3

THIRD TERM

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)} \right] G_{16}$$ 49

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)} \right] G_{17}$$ 50

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)} \right] G_{18}$$ 51

Where $\boxed{+\left(a''_{16}\right)^{(2)}(T_{17},t)}$, $\boxed{+\left(a''_{17}\right)^{(2)}(T_{17},t)}$, $\boxed{+\left(a''_{18}\right)^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

And $\boxed{+\left(a''_{13}\right)^{(1,1,1)}(T_{14},t)}$, $\boxed{+\left(a''_{14}\right)^{(1,1,1)}(T_{14},t)}$, $\boxed{+\left(a''_{15}\right)^{(1,1,1)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

SECOND TERM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\right]T_{13}$$ 52

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\right]T_{14}$$ 53

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\right]T_{15}$$ 54

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detritions coefficient for category 1, 2 and 3

FIRST TERM

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\right]G_{13}$$ 55

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\right]G_{14}$$ 56

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\right]G_{15}$$ 57

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

THIRD TERM

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1)}(G,t)}\right]T_{16}$$ 58

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1)}(G,t)}\right]T_{17}$$ 59

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1)}(G,t)}\right]T_{18}$$ 60

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$ are second detritions coefficients for category 1,2 and 3

Uncertainty principle

In quantum mechanics, the uncertainty principle is any of a variety of mathematical inequalities asserting a fundamental lower bound on the precision with which certain pairs of physical properties of a particle, such as position x and momentum p, can be simultaneously known. The more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa The original heuristic argument that such a limit should exist was given by Werner Heisenberg in 1927. A more formal inequality relating the standard deviation of position σx and the standard deviation of momentum σp was derived by Kennard later that year (and independently by Weyl in 1928),

$$\sigma_x \sigma_p \geq \frac{\hbar}{2},$$

Where ħ is the reduced Planck constant.

Historically, the uncertainty principle has been confused with a somewhat similar effect in physics, called the observer effect which notes that measurements of certain systems cannot be made without affecting the systems. Heisenberg himself offered such an observer effect at the quantum level (see below) as a physical "explanation" of quantum uncertainty. However, it has since become clear that quantum uncertainty is inherent in the properties of all wave-like systems, and that it arises in quantum mechanics simply due to the matter wavenature of all quantum objects. Thus, the uncertainty principle actually states a fundamental property of quantum systems, and is not a statement about the observational success of current technology

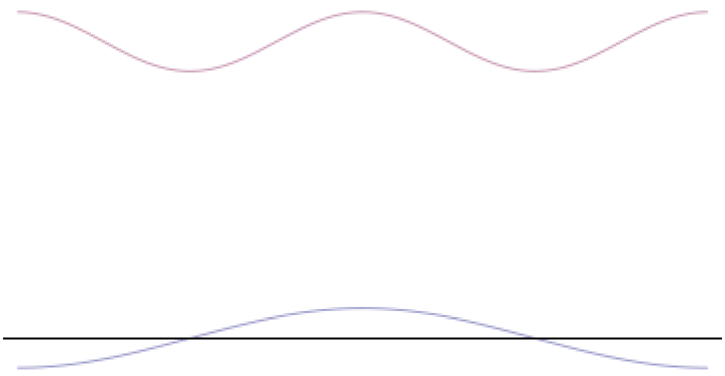
Mathematically, the uncertainty relation between position and momentum arises because the expressions of the wavefunction in the two corresponding bases are Fourier transforms of one another (i.e., position and momentum are conjugate variables). A similar tradeoff between the variances of Fourier conjugates arises wherever Fourier analysis is needed, for example in sound waves. A pure tone is a sharp spike at a single frequency. Its Fourier transform gives the shape of the sound wave in the time domain, which is a completely delocalized sine wave. In quantum mechanics, the two key points are that the position of the particle takes the form of a matter wave, and momentum is its Fourier conjugate, assured by the de Broglie relation $p = \hbar k$, where k is the wave number.

In the mathematical formulation of quantum mechanics, any pair of non-commuting self-adjoint operators representing observables are subject to similar uncertainty limits. An eigenstate of an observable represents the state of the wavefunction for a certain measurement value (the eigenvalue). For example, if a measurement of an observable A is taken then the system is in a particular eigenstate Φ of that observable. The particular eigenstate of the observable A may not be an eigenstate of another observable B. If this is so, then it does not have a single associated measurement as the system is not in an eigenstate of the observable

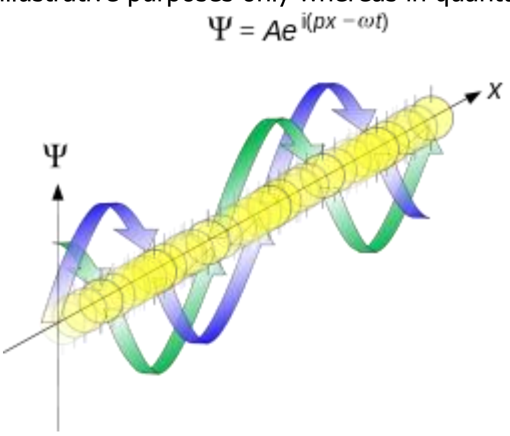
<u>THE UNCERTAINTY PRINCIPLE</u>:

The uncertainty principle can be interpreted in either the wave mechanics or matrix mechanics formalisms of quantum mechanics. Although the principle is more visually intuitive in the wave mechanics formalism, it was first derived and is more easily generalized in the matrix mechanics formalism. We will attempt to motivate the principle in the two frameworks.

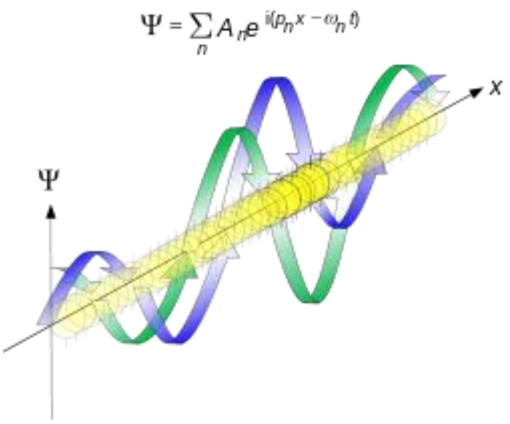
<u>Wave mechanics interpretation</u>



The superposition of several plane waves. The wave packet becomes increasingly localized with the addition of many waves. The Fourier transform is a mathematical operation that separates a wave packet into its individual plane waves. Note that the waves shown here are real for illustrative purposes only whereas in quantum mechanics the wave function is generally complex.



Plane wave



Wave packet

Propagation of de Broglie waves in 1d - real part of the complex amplitude is blue, imaginary part is green. The probability (shown as the colouropacity) of finding the particle at a given point x is spread out like a waveform, there is no definite position of the particle. As the amplitude increases above zero the curvature decreases, so the decreases again, and vice versa - the result are alternating amplitude: a wave.

According to the de Broglie hypothesis, every object in our Universe is a wave, a situation which gives rise to this phenomenon. The position of the particle is described by a wave function $\Psi(x,t)$. The time-independent wave function of a single-moded plane wave of wave number k0 or momentum p0 is

$$\psi(x) \propto e^{ik_0x} = e^{ip_0x/\hbar}$$

The Born rule states that this should be interpreted as a probability density function in the sense that the probability of finding the particle between a and b is

$$\mathrm{P}[a \leq X \leq b] = \int_a^b |\psi(x)|^2 \,\mathrm{d}x.$$

In the case of the single-moded plane wave, $|\psi(x)|^2$ is a uniform distribution. In other words, the particle position is extremely uncertain in the sense that it could be essentially anywhere along the wave packet. However, consider a wave function that is a sum of many waves. We may write this as

$$\psi(x) \propto \sum_n A_n e^{ip_nx/\hbar},$$

Where An represents the relative contribution of the mode pn to the overall total. The figures to the right show how with the addition of many plane waves, the wave packet can become more localized. We may take this a step further to the continuum limit, where the wave function is an integral over all possible modes

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) \cdot e^{ipx/\hbar}\, dp,$$

With $\phi(p)$ representing the amplitude of these modes and is called the wave function in momentum space. In mathematical terms, we say that $\phi(p)$ is the Fourier transforms of $\psi(x)$ and that x and p are conjugate variables. Adding together all of these plane waves comes at a cost, namely the momentum has become less precise, having become a mixture of waves of many different momenta.

One way to quantify the precision of the position and momentum is the standard deviation σ. Since $|\psi(x)|^2$ is a probability density function for position, we calculate its standard deviation.

We improved the precision of the position, i.e. reduced σx, by using many plane waves, thereby weakening the precision of the momentum, i.e. increased σp. Another way of stating this is that σx and σp has an inverse relationship or are at least bounded from below. This is the uncertainty principle, the exact limit of which is the Kennard bound. Click the show button below to see a semi-formal derivation of the Kennard inequality using wave mechanics.

Matrix mechanics interpretation

In matrix mechanics, observables such as position and momentum are represented by self-adjoint operators. When considering pairs of observables, one of the most important quantities is the commutator. For a pair of operators $\hat{A}$ and $\hat{B}$, we may define their commutator as

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.$$

In the case of position and momentum, the commutator is the canonical commutation relation

$$[\hat{x}, \hat{p}] = i\hbar.$$

The physical meaning of the non-commutativity can be understood by considering the effect of the commutator on position and momentum eigenstates. Let $|\psi\rangle$ be a right eigenstate of position with a constant eigenvalue x0. By definition, this means that $\hat{x}|\psi\rangle = x_0|\psi\rangle.$ Applying the commutator to $|\psi\rangle$ yields

$$[\hat{x}, \hat{p}]|\psi\rangle = (\hat{x}\hat{p} - \hat{p}\hat{x})|\psi\rangle = (\hat{x} - x_0\hat{I}) \cdot \hat{p}\,|\psi\rangle = i\hbar|\psi\rangle,$$

where $\hat{I}$ is simply the identity operator. Suppose for the sake of proof by contradiction that $|\psi\rangle$ is also a right eigenstate of momentum, with constant eigenvalue p0. If this were true, then we could write

$$(\hat{x} - x_0\hat{I}) \cdot \hat{p}\,|\psi\rangle = (\hat{x} - x_0\hat{I}) \cdot p_0\,|\psi\rangle = (x_0\hat{I} - x_0\hat{I}) \cdot p_0\,|\psi\rangle = 0.$$

On the other hand, the canonical commutation relation requires that

$$[\hat{x}, \hat{p}]|\psi\rangle = i\hbar|\psi\rangle \neq 0.$$

This implies that no quantum state can be simultaneously both a position and a momentum eigenstate. When a state is measured, it is projected onto an eigenstate in the basis of the observable. For example, if a particle's position is measured, then the state exists at least momentarily in a position eigenstate. However, this means that the state is not in a momentum eigenstate but rather exists as a sum of multiple momentum basis eigenstates. In other words the momentum must be less precise. The precision may be quantified by the standard deviations, defined by

$$\sigma_x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$$

$$\sigma_p = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2}.$$

As with the wave mechanics interpretation above, we see a tradeoff between the precisions of the two, given by the uncertainty principle.

Robertson-Schrödinger uncertainty relations

The most common general form of the uncertainty principle is the Robertson uncertainty relation. For an arbitrary Hermitian operator $\hat{\mathcal{O}}$, we can associate a standard deviation

$$\sigma_{\mathcal{O}} = \sqrt{\langle \hat{\mathcal{O}}^2 \rangle - \langle \hat{\mathcal{O}} \rangle^2},$$

Where the brackets $\langle \mathcal{O} \rangle$ indicate an expectation value. For a pair of operators $\hat{A}$ and $\hat{B}$, we may define their commutator as

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A},$$

In this notation, the Robertson uncertainty relation is given by

$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|.$$

he Robertson uncertainty relation immediately follows from a slightly stronger inequality, the Schrödinger uncertainty relation,

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2}\langle\{\hat{A},\hat{B}\}\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle\right)^2 + \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2,$$

Where we have introduced the anticommutator,

$$\{\hat{A},\hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}.$$

Since the Robertson and Schrödinger relations are for general operators, the relations can be applied to any two observables to obtain specific uncertainty relations. A few of the most common relations found in the literature are given below.

For position and linear momentum, the canonical commutation relation $[\hat{x},\hat{p}] = i\hbar$ implies the Kennard inequality from above:

$$\sigma_x\sigma_p \geq \frac{\hbar}{2}$$

For two orthogonal components of the total angular momentum operator of an object:

$$\sigma_{J_i}\sigma_{J_j} \geq \frac{\hbar}{2}\left|\langle J_k\rangle\right| \quad ,$$

where i, j, k are distinct and Ji denotes angular momentum along the xi axis. This relation implies that only a single component of a system's angular momentum can be defined with arbitrary precision, normally the component parallel to an external (magnetic or electric) field. Moreover, for $[J_x, J_y] = i\hbar\epsilon_{xyz}J_z$, a choice $\hat{A} = J_x, \quad \hat{B} = J_y$, in angular momentum multiplets, ψ = |j, m ⟩, bounds the Casimir invariant (angular momentum squared, $\langle J_x^2 + J_y^2 + J_z^2\rangle$) from below and thus yields useful constraints such as j (j+1) ≥ m (m + 1), and hence j ≥ m, among others.

In non-relativistic mechanics, time is privileged as an independent variable. Nevertheless, in 1945, L. I. Mandelshtam and I. E. Tamm derived a non-relativistic time-energy uncertainty relation, as follows For a quantum system in a non-stationary state ψ and an observable B represented by a self-adjoint operator $\hat{B}$, the following formula holds:

$$\sigma_E \frac{\sigma_B}{\left|\frac{\mathrm{d}\langle\hat{B}\rangle}{\mathrm{d}t}\right|} \geq \frac{\hbar}{2},$$

Where σ_E is the standard deviation of the energy operator in the state ψ, σ_B stands for the standard deviation of B Although the second factor in the left-hand side has dimension of time, it is different from the time parameter that enters Schrödinger equation. It is a lifetime of the state ψ with respect to the observable B.

IT IS TO BE NOTED THAT DESPITE THE UNIVERSALITY OF THE THEORY, SAY OF NEWTON, THERE EXISTS WHAT COULD BE CALLED AS "TOTAL GRAVITY" JUST BECAUSE THERE IS CONSTANT MAINTAINANCE OF BALANCE IN ACCOUNTS IN THE BANK IT DOES NOT MEAN THAT THERE DOES NOT EXIST ANY OPERATIONS, NOR IS THERE NO TOTAL ASSETS OR LIABILITIES. IN FACT LIKE IN A CLOSED ECONOMY IT DOES. SO, WHEN WE SAY THE FIRST TERMS OF EFE ARE CLASSIDFIED IN TO VARIOUS CATEGORIES WE

REFER TO THE FACT THAT VARIOUS SYSTEMS ARE UNDER CONSIDERATION AND THEY OFCOURSE SATISFY GTR. THE SAME EXPLANTION HOLDS GOOD IN THE STARTIFICATION OF THE HEISENBERG'S PRINCIPLE OF UNCERTAINTY. FIRST, WE DISCUSS THE EQUALITY CASE. WE TRANSFER THETERM REPRESENTATIVE OF POSITION OF PARTICLE OR THE ONE CONSTITUTIVE OF MONEMTUM TO THE OTHER SIDE AND THE RELATIONSHIP THAT EXISTS NOW BETWIXT "POSITION" AND MOMENTUM IS THAT THE INVERSE OF ONE IS BEING "SUBTRACTED "FROM THE OTHER. THIS APPARANTELY MEANS THAT ONE TERM IS BEITAKEN OUT FROM THE OTHER. THERE MAY OR MIGHT NOT BE TIME LAG. THAT DOES NOT MATTYER IN OPUR CALCULATION. THE EQUATIONS REPRESENT AND CONSTITUTE THE GLOBALISED EQUATIONS WHICH IS BASED ON THE SIMPLE MATTER OF ACCENTUATION AND DISSSIPATION. IN FACT THE FUNCTIONAL FORMS OF ACCENTUATION AND DISSIPATION TERMS THEMSELVES ARE DESIGNATIVE OF THE FACT THAT THERE EXISTS A LINK BETWEEN THE VARIOUS THEORIES GALILEAN, PLATONIC, MENTAL, GTR, STR, QM.QFT, AND QUANTUM GRAVITY. FINALLY. I AN SERIES OF PAPER WE SHALL BUILD UP THE STRUCTURE TOWARDS THE END OF CONSUMMATION OF THE UNIFICATION OF THE THEORIES. THAT OINE THEORY IS RELATED TO ANOTHER IS BEYOND DISPUTE AND WE TAKE OFF FROM THAT POINT TOWARDS OUR MISSION.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

FIRST TERM AND SECOND TERM IN EFE :

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G_{13} : CATEGORY ONE OF THE FIRST TERM IN EFE

G_{14} : CATEGORY TWO OF THE FIRST TERM IN EFE

G_{15} : CATEGORY THREEOF FIRST TERM IN EFE

T_{13} : CATEGORY ONE OF SECOND TERM IN EFE

T_{14} : CATEGORY TWO OF THE SECOND TERM IN EFE

T_{15} : CATEGORY THREE OF SECOND TERM IN EFE

THIRD TERM AND FOURTH TERM OF EFE: NOTE FOURTH TERM ON RHS IS REMOVED FROM THE THIRD TERM

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G_{16} : CATEGORY ONE OF THIRD TERM OF EFE

G_{17} : CATEGORY TWO OF THIRD TERM OF EFE

G_{18} : CATEGORY THREE OF THIRD TERM OF EFE

T_{16} : CATEGORY ONE OF FOURTH TERM OF EFE 1

T_{17} : CATEGORY TWO OF FOURTH TERM OF EFE

T_{18} : CATEGORY THREE OF FOURTH TERM OF EFE

HEISENBERG'S UNCERTAINTY PRINCIPLE: $\sigma_x \sigma_p \geq \frac{\hbar}{2},$

NOTE THE FIRST TERM IS INVERSELY PROPORTIONAL TO THE SECOND TERM. TAKE THE EQUALITY CASE. THIS LEADS TO SUBTRACTION OF THE SECOND TERM ON THE RHS FROM THE FIRST TERM THIS WWE SHALL MODEL AND ANNEX WITH EFE. DESPITE HUP HOLDING GOOD FOR ALL THE SYSTEMS, WE CAN CLASSIFY THE SYSTEMS STUDIED AND NOTE THE REGISTRATIONS IN EACH SYSTEM. AS SAID EARLIER THE FIRST TERM VALUE OF SOME SYSTEM, THE SECOND TERM VALUE OF SOME OTHER DIFFERENTIATION CARRIED OUT BASED ON PARAMETRICIZATION

==

G_{20} : CATEGORY ONE OF FIRST TERM ON HUP

G_{21} : CATEGORY TWO OF FIRST TERM OF HUP

G_{22} : CATEGORY THREE OF FIRST TERM OF HUP

T_{20} : CATEGORY ONE OF SECOND TERM OF HUP

T_{21} : CATEGORY TWO OF SECOND TERM OF UCP

T_{22} : CATEGORY THREE OF SECOND TERM OF HUP

ACCENTUATION COEFFCIENTS:OF HOLISTIC SYSTEM EFE-HUP SYSTEM

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: $(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$

===

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$, $(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$

FIRST TERM OF EFE- SECOND TERM OF EFE:GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) \right] G_{14} \quad 1$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) \right] G_{15} \quad 2$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t) \right] T_{13} \quad 3$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) \right] T_{14} \quad 4$$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G,t)\right]T_{15}$ 5

$+(a_{13}^{''})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b_{13}^{''})^{(1)}(G,t) =$ First detritions factor 6

7

GOVERNING EQUATIONS:THIRD TERM OF EFE AND FOURTH TERM OF EFE:

The differential system of this model is now

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17},t)\right]G_{16}$ 8

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17},t)\right]G_{17}$ 9

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17},t)\right]G_{18}$ 10

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}((G_{19}),t)\right]T_{16}$ 11

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}((G_{19}),t)\right]T_{17}$ 12

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}((G_{19}),t)\right]T_{18}$ 13

$+(a_{16}^{''})^{(2)}(T_{17},t) =$ First augmentation factor 14

$-(b_{16}^{''})^{(2)}((G_{19}),t) =$ First detritions factor 15

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GOVERNING EQUATIONS:OF THE FIRST TERM AND SECOND TERM OF HUP:NOTE THAT FSECOND TERM (INVERSE THEREOF) IS SUBTRACTED FROM THE FIRST TERM ,WHICH MEANS THE AMOUNT IS REMOVED FOR INFINITE SYSTEMS IN THE WORLD. LAW OFCOURSE HOLDS FOR ALL THE SYSTEMS BY THIS METHIODLOGY WE GET THE VALUE OF THE FIRST TERM AS WELL AS THE SECOND TERM

$$\sigma_x\sigma_p \geq \frac{\hbar}{2},$$

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The differential system of this model is now

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21},t)\right]G_{20}$ 17

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21},t)\right]G_{21}$ 18

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21},t)\right]G_{22}$ 19

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23},t)\right]T_{20}$ 20

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23},t)\right]T_{21}$ 21

$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t) \right] T_{22}$ 22

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor 23

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor 24

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GOVERNING EQUATIONS OF THE HOLISTIC SYSTEM FOUR TERMS OF EFE AND TWO TERMS OF HUP: 26

$$\sigma_x \sigma_p \geq \frac{\hbar}{2},$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

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$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)} \right] G_{13}$ *27*

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,)}(T_{21}, t)} \right] G_{14}$ *28*

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,)}(T_{21}, t)} \right] G_{15}$ *29*

Where $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3 *30*

$\boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

31

$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2,)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,)}(G_{23}, t)} \right] T_{13}$ 32

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2,)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,)}(G_{23}, t)} \right] T_{14}$ 33

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2,)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23}, t)} \right] T_{15}$ *34*

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3 *35*

$\boxed{-(b''_{16})^{(2,2,)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23}, t)}$ are third detrition coefficients for category 1, 2 and 3

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}\right]G_{16}$ 36

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}\right]G_{17}$ 37

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}\right]G_{18}$ 38

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 39

And $\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

40

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}\right]T_{16}$ 41

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}\right]T_{17}$ 42

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}\right]T_{18}$ 43

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 44

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

45

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}\right]G_{20}$ 46

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}\right]G_{21}$ 47

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}\right]G_{22}$ 48

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3 49

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

50

$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23},t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \right] T_{20}$ *51*

$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23},t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \right] T_{21}$ *52*

$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23},t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \right] T_{22}$ *53*

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ are first detrition coefficients for category 1, 2 and 3 *54*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ are third detrition coefficients for category 1,2 and 3

55

Where we suppose *56*

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$ *57*

$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

<u>Definition of</u> $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G,t) \le \ (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14},t) = (p_i)^{(1)}$ 58

$\lim_{G \to \infty} (b''_i)^{(1)} (G,t) = \ (r_i)^{(1)}$

<u>Definition of</u> $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants
and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 59

$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)} t}$ 60

$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,t)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)} t}$ 61

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14},t)$ and $(a''_i)^{(1)}(T_{14},t)$. (T'_{14},t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)} \right]$. It is to be noted that $(a''_i)^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14},t)$, the first augmentation coefficient WOULD BE absolutely continuous. 62

<u>Definition of</u> $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 63

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 64

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants 65

$(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ 66

satisfy the inequalities 67

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$ 68

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose 69

(F) $(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$ 70

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded. 71

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$: 72

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)}$$ 73

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)}$$ 74

(H) $\lim_{T_2 \to \infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$ 75

$\lim_{G \to \infty}(b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)}$ 76

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 77

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 78

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$$ 79

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$$ 80

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient attributable to would be absolutely continuous. 81

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 82

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 83

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 84

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants

$(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1$ 85

$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1$ 86

Where we suppose 87

(J) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0,$ 88

$i, j = 20,21,22$

(K) The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (r_i)^{(3)}$:

$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$

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(L) $lim_{T_2 \to \infty}(a_i'')^{(3)}\,(T_{21}, t) = (p_i)^{(3)}$ 90

$\lim_{G \to \infty}(b_i'')^{(3)}\,(G_{23}, t) = \ (r_i)^{(3)}$ 91

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 92

$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$ 93

$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$ 94

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient would be absolutely continuous. 95

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 96

(M) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants 97

$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$ 98

satisfy the inequalities 99

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$ 100

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$ 101

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Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 103

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \ (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \ (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)$, $T_i(0)$

$G_i(t) \leq \ (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq \ (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

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$G_i(t) \leq \ (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq \ (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Proof: 108

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 109

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 110

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 111

By 112

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{13}(s_{(13)})\right] ds_{(13)}$

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{14}(s_{(13)})\right] ds_{(13)}$ 113

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}\left(T_{14}(s_{(13)}), s_{(13)}\right)\right) G_{15}(s_{(13)})\right] ds_{(13)}$ 114

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{13}(s_{(13)})\right] ds_{(13)}$ 115

$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$ 116

$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$ 117

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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Proof: 119

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0, \; T_i(0) = T_i^0, \; G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$ 120

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 121

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 122

By 123

$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$

$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$ 124

$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$ 125

$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$ 126

$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$ 127

$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$ 128

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 129

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0, \; T_i(0) = T_i^0, \; G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$ 130

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 131

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 132

By 133

$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$

$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$ 134

$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$ 135

$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$ 136

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}\left(G(s_{(20)}), s_{(20)}\right) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$ 137

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}\left(G(s_{(20)}), s_{(20)}\right) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$ 138

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

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(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 141

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 142

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$ 143

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 144

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} =$$ 145

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that 146

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$ 147

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 148

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that 149

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0} \right)} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 150

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It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 152

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)}$$ 153

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$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i into itself 155

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 156

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 157

Definition of $\tilde{G}, \tilde{T}$:

$$\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G,T)$$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)} - G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t} \le \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$ 158

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 159

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 160

From GLOBAL EQUATIONS it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for } t > 0$$

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Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 162

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 163

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 164

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 165

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 16

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)}$$ 7

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$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i into itself 169

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 170

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right)=$$ 171

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 172

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$:

$$\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$$

It results 173

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a_{16}'')^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)|\;e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 174

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$ 175

$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a_{16}')^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+\right.$$

$$\left.(\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)},(T_{19})^{(1)};\;(G_{19})^{(2)},(T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 176

Remark 1: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 177

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $\mathrm{G}_i(\mathrm{t}) = 0$ and $\mathrm{T}_i(\mathrm{t}) = 0$ 178

From GLOBAL EQUTIONS it results

$$\mathrm{G}_i(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)}-(a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}),s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$$

$\mathrm{T}_i(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0$ for $\mathrm{t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1, \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 179

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

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In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 181

Remark 5: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}((G_{19})(\mathrm{t}),\mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 182

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $\mathrm{t} > \mathrm{t}_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(\mathrm{t}),\mathrm{t}) < \varepsilon_2, \mathrm{T}_{16}(\mathrm{t}) > (m)^{(2)}$

Then $\frac{\mathrm{dT}_{17}}{\mathrm{dt}} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 \mathrm{T}_{17}$ which leads to 183

$\mathrm{T}_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - \mathrm{e}^{-\varepsilon_2 \mathrm{t}}) + \mathrm{T}_{17}^0 \mathrm{e}^{-\varepsilon_2 \mathrm{t}}$ If we take t such that $\mathrm{e}^{-\varepsilon_2 \mathrm{t}} = \frac{1}{2}$ it results

$\mathrm{T}_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(\mathrm{t}),\mathrm{t}\right) = (b_{18}')^{(2)}$ 184

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 186

$(\widehat{\mathrm{P}}_{20})^{(3)}$ and $(\widehat{\mathrm{Q}}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)}$$ 187

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$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself 189

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 190

$$d\left(\left((G_{23})^{(1)},(T_{23})^{(1)}\right),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 191

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}$ $:\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right)=\mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 192

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$ 193

$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+\right.$$
$$\left.(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 194

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 195

From 19 to 24 it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 196

Remark 3: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 197

Remark 5: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 198

Definition of $(m)^{(3)}$ and ε_3 :

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Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 200

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

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Behavior of the solutions 202

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$

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Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$:

(b) By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 204

By $(\bar{\nu}_1)^{(1)} > 0\,, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0\,, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}\,, (m_2)^{(1)}\,, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 205

(c) If we define $(m_1)^{(1)}\,, (m_2)^{(1)}\,, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}\,, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 206

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\,, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$ 207

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution satisfies the inequalities 208

$$G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined $\frac{1}{(m_1)^{(1)}}$

$G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$

$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}\left((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)}\right)}\left[e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right.$ 209
$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}\left((S_1)^{(1)} - (a'_{15})^{(1)}\right)}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$

$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{\left((R_1)^{(1)} + (r_{13})^{(1)}\right)t}}$ 210

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{\left((R_1)^{(1)} + (r_{13})^{(1)}\right)t}$ 211

$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}\left((R_1)^{(1)} - (b'_{15})^{(1)}\right)}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le$ 212

$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}\left((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)}\right)}\left[e^{\left((R_1)^{(1)} + (r_{13})^{(1)}\right)t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 213

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

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Behavior of the solutions 215

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 216

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \leq -(\sigma_1)^{(2)}$ 217

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \leq -(\tau_1)^{(2)}$ 218

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 219

By $(\nu_1)^{(2)} > 0, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots 220

(e) of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 221

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 222

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 223

By $(\bar{\nu}_1)^{(2)} > 0, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 224

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 225

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 226

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 227

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 228

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \ \boldsymbol{if} \ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 229

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if} \ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)},$ 230

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 231

and analogously 232

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if} \ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, $\boldsymbol{if}$ $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, $\boldsymbol{if}$ $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 234

Then the solution satisfies the inequalities 235

$$G_{16}^0 e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined 236

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$ 237

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)}-(p_{16})^{(2)}-(S_2)^{(2)})}\left[e^{((S_1)^{(2)}-(p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)}-(a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$ 238

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)}+(r_{16})^{(2)})t}}$$ 239

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)}+(r_{16})^{(2)})t}$$ 240

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)}-(b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$ 241

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$ 242

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 243

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 244

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 245

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

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Behavior of the solutions 247

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 248

(b) By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}$, $(\mu_2)^{(3)}$:- 249

(c) If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}$, $(\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)},\ \boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}\ ,\boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)},$

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 250

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}\ ,\boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

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Then the solution satisfies the inequalities

$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined

$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$ 252

$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)} \left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \right.$ 253
$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}\left((S_1)^{(3)} - (a_{22}')^{(3)}\right)} \left[e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t} \right] + G_{22}^0 e^{-(a_{22}')^{(3)}t} \right)$

$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}}$ 254

$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}$ 255

$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}\left((R_1)^{(3)} - (b_{22}')^{(3)}\right)} \left[e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t} \right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq$ 256

$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}\left((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)}\right)} \left[e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 257

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

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FROM GLOBAL EQUATIONS WE OBTAIN: 259

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

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In the same manner , we get

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 261

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 262

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

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: From GLOBAL EQUATIONS we obtain 264

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right) - (a''_{17})^{(2)}(T_{17},t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 265

It follows 266

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 267

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 268

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 269

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 270

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 271

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 272

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 273

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

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Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem. 275

Particular case : 276

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

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FROM GLOBAL EQUATIONS WE OBTAIN: 278

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 279

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2+(\sigma_2)^{(3)}\nu^{(3)}-(a_{20})^{(3)}\right)\le\frac{d\nu^{(3)}}{dt}\le-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2+(\sigma_1)^{(3)}\nu^{(3)}-(a_{20})^{(3)}\right)$$

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From which one obtains

(a) For $0<(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\nu_1)^{(3)}<(\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t)\ge\frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)}=\frac{(\nu_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)}\le\nu^{(3)}(t)\le(\nu_1)^{(3)}$

In the same manner , we get 281

$$\nu^{(3)}(t)\le\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)}=\frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

<u>Definition of</u> $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)}\le\nu^{(3)}(t)\le(\bar{\nu}_1)^{(3)}$

(b) If $0<(\nu_1)^{(3)}<(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}<(\bar{\nu}_1)^{(3)}$ we find like in the previous case, 282

$$(\nu_1)^{(3)}\le\frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}\le\nu^{(3)}(t)\le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}\le(\bar{\nu}_1)^{(3)}$$

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(c) If $0<(\nu_1)^{(3)}\le(\bar{\nu}_1)^{(3)}\le(\nu_0)^{(3)}=\frac{G_{20}^0}{G_{21}^0}$, we obtain 284

$$(\nu_1)^{(3)}\le\nu^{(3)}(t)\le\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}\le(\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(3)}(t)$:-

$$(m_2)^{(3)}\le\nu^{(3)}(t)\le(m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t)=\frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

<u>Definition of</u> $u^{(3)}(t)$:-

$$(\mu_2)^{(3)}\le u^{(3)}(t)\le(\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t)=\frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

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We can prove the following

Theorem 3: If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation 25 are satisfied , then the system

If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions 286

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 287

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 288

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 289

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 290

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions 291

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied , then the system

$(a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right] G_{13} = 0$ 292

$(a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right] G_{14} = 0$ 293

$(a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right] G_{15} = 0$ 294

$(b_{13})^{(1)} T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ] T_{13} = 0$ 295

$(b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ] T_{14} = 0$ 296

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 297

has a unique positive solution , which is an equilibrium solution for the system 298

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 299

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 300

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 301

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 302

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 303

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 304

has a unique positive solution , which is an equilibrium solution for 305

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 306

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 230
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$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 308

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 309

$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0$ 310

$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0$ 311

has a unique positive solution , which is an equilibrium solution for 312

Proof: 313

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

314

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

315

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Definition and uniqueness of T_{14}^{*} :- 316

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)} G_{14}}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)} G_{14}}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :- 317

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(\mathrm{T}_{17}^*) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)} \mathrm{G}_{17}}{[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(\mathrm{T}_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)} \mathrm{G}_{17}}{[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(\mathrm{T}_{17}^*)]}$$ 318

Definition and uniqueness of T_{21}^* :- 319

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$$

(b) By the same argument, the equations (SOLUTIONAL) admit solutions G_{13}, G_{14} if 320

$$\varphi(G) = (b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b_{13}')^{(1)}(b_{14}'')^{(1)}(G) + (b_{14}')^{(1)}(b_{13}'')^{(1)}(G)\right] + (b_{13}'')^{(1)}(G)(b_{14}'')^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(c) By the same argument, the equations (SOLUTIONAL) admit solutions G_{16}, G_{17} if 321

$$\varphi(G_{19}) = (b_{16}')^{(2)}(b_{17}')^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b_{16}')^{(2)}(b_{17}'')^{(2)}(G_{19}) + (b_{17}')^{(2)}(b_{16}'')^{(2)}(G_{19})\right] + (b_{16}'')^{(2)}(G_{19})(b_{17}'')^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 322

(d) By the same argument, the equations(SOLUTIONAL) admit solutions G_{20}, G_{21} if 323

$$\varphi(G_{23}) = (b_{20}')^{(3)}(b_{21}')^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b_{20}')^{(3)}(b_{21}'')^{(3)}(G_{23}) + (b_{21}')^{(3)}(b_{20}'')^{(3)}(G_{23})\right] + (b_{20}'')^{(3)}(G_{23})(b_{21}'')^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

Finally we obtain the unique solution 324

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]}$, $G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$

$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]}$, $T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 325

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(\mathrm{T}_{17}^*) = 0$ and 326

$\mathrm{G}_{16}^* = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^*}{[(\mathrm{a}'_{16})^{(2)} + (\mathrm{a}''_{16})^{(2)}(\mathrm{T}_{17}^*)]}$, $\mathrm{G}_{18}^* = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^*}{[(\mathrm{a}'_{18})^{(2)} + (\mathrm{a}''_{18})^{(2)}(\mathrm{T}_{17}^*)]}$ 327

$\mathrm{T}_{16}^* = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^*}{[(\mathrm{b}'_{16})^{(2)} - (\mathrm{b}''_{16})^{(2)}((G_{19})^*)]}$, $\mathrm{T}_{18}^* = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^*}{[(\mathrm{b}'_{18})^{(2)} - (\mathrm{b}''_{18})^{(2)}((G_{19})^*)]}$ 328

Obviously, these values represent an equilibrium solution 329

Finally we obtain the unique solution 330

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]}$, $G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$

$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]}$, $T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 331

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. 332

Proof:_Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (GLOBAL)and neglecting the terms of power 2, we obtain 333

$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$ 334

$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$ 335

$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$ 336

$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right)$ 337

$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right)$ 338

$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right)$ 339

If the conditions of the previous theorem are satisfied and if the functions $(\mathrm{a}''_i)^{(2)}$ and $(\mathrm{b}''_i)^{(2)}$ 340
Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

_Denote 341

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$\mathrm{G}_i = \mathrm{G}^*_i + \mathbb{G}_i \qquad , \mathrm{T}_i = \mathrm{T}^*_i + \mathbb{T}_i$ 342

$\frac{\partial(a''_{17})^{(2)}}{\partial \mathrm{T}_{17}}(\mathrm{T}^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial \mathrm{G}_j}\left((G_{19})^*\right) = s_{ij}$ 343

taking into account equations(GLOBAL) and neglecting the terms of power 2, we obtain 344

$\frac{\mathrm{d}\mathbb{G}_{16}}{\mathrm{dt}} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}\mathrm{G}^*_{16}\mathbb{T}_{17}$ 345

$\frac{\mathrm{d}\mathbb{G}_{17}}{\mathrm{dt}} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}\mathrm{G}^*_{17}\mathbb{T}_{17}$ 346

$\frac{\mathrm{d}\mathbb{G}_{18}}{\mathrm{dt}} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}\mathrm{G}^*_{18}\mathbb{T}_{17}$ 347

$\frac{\mathrm{d}\mathbb{T}_{16}}{\mathrm{dt}} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}\mathrm{T}^*_{16}\mathbb{G}_j\right)$ 348

$\frac{\mathrm{d}\mathbb{T}_{17}}{\mathrm{dt}} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right)$ 349

$\frac{\mathrm{d}\mathbb{T}_{18}}{\mathrm{dt}} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right)$ 350

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$ 351
Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically sta

_Denote 352

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G^*_i + \mathbb{G}_i \qquad , T_i = T^*_i + \mathbb{T}_i$

$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$

Then taking into account equations (GLOBAL AND CONCATENATED) and neglecting the terms of power 2, we obtain 353

$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21}$ 354

$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21}$ 355

$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21}$ 356

$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right)$ 357

$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right)$ 358

$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right)$ 359

The characteristic equation of this system is 360

$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$

$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)\right]$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(14)}T_{14}^{*}\right)$

$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^{*} + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^{*}\right)$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(13)}T_{13}^{*}\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$

$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^{*} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^{*}\right)$

$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^{*} + (b_{14})^{(1)}s_{(13),(15)}T_{13}^{*}\right)\} = 0$

+

$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$

$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}_{17}^{*} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}_{16}^{*}\right)\right]$

$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}_{17}^{*} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}_{17}^{*}\right)$

$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}_{16}^{*} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}_{17}^{*}\right)$

$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}_{17}^{*} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}_{16}^{*}\right)$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$

$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$

$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}_{17}^{*} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}_{16}^{*}\right)$

$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}_{17}^{*} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}_{16}^{*}\right)\} = 0$

+

$$((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)})\{((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)})$$

$$\left[\left(((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)})(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)})s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)})(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)})s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(((\lambda)^{(3)})^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(((\lambda)^{(3)})^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(((\lambda)^{(3)})^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)})\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)})s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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