Construction of Norm On Class of (123) - Avoiding Pattern of Aunu Permutation Pattern

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Abstract- This paper shows the application of AUNU permutation pattern on norm space. Were the sequence of 123-avoiding pattern was use to construct a vector space and result follows, we also used constructed vector space to construct a norm space and normed space was also defined.

Index Terms- AUNU Groups, AUNU Patterns, Vector space, Normed Vector Space

I. Introduction

Norm and Normed linear space has so many Application in different area of mathematics, we begin with a quotation from Stefan Banach's thesis (1922, pg. 134-136) submitted to the university of Lwow in June 1920. He stated the modern definition of vector space axiomatically, given in the vector space. Friedrich Riesz, thesis (1910) no2 on fixed point theorems follow the idea of M. Frechet and introduced the concept of a norm on a vector space, as a function which take a vector into a real. Eduard Helly, (1920) and Hans Hahn (1922) on a problem of moment work with general sequence space endowed with general norm, and observed that this norm satisfied some certain properties and defined normed space axiomatically. On continuous transformation of vector space. N. Wiener thesis (1922) defined normed space. T. K. Samanta *et al* (2010) Proved the completeness of a finite dimensional cone normed linear space and study the cone convergence with respect to the cone norms. Hendra Gunawan and M. Mashadi (2000) Provide a simple way to derive an $(n-1)$ normed from the n- normed and realized that any n-normed space is an $(n-1)$ n-normed space. A fixed points theorem for some n-Banach space was proved. Benedict's Banes *et al* (2018) Use the binomial inequalities to provide alternative method of quasi-normed linear space. The establishment of the subspace, quasi-product normed of the quasi normed linear space was observed; Finally, the researchers show that the quasi- normed linear space admits quasi Banach's dilation which is lipschitzian mapping. Anlrban Kundu *et al* (2019) study a topological structure of 2-normed space from the view point of topological vector spaces. A separating family of semi norms is induced from a given 2- norms, and the criteria for metrizability and a necessary and sufficient for normability of 2-normed space has been reduced. With this, our research is going to use the sequence of AUNU Permutation Pattern to provide alternative method of defining Vector Space, Normed Linear Space and Normed Space.

II. Some Basic Definitions

2.1. Vector space.

Let $\mathbb{F}$ denote the set of real or complex numbers, and V be a vector space over $\mathbb{F}$. Then for any $x, y \in V$. The following axioms of vector space hold;

. 1. $x + y \in V$ whenever $x, y \in V$;
2. $x + y = y + x$ for all $x, y \in V$;
3. There exists a unique element in V, denoted by 0, such that $x + 0 = 0 + x = x$ for all $x \in V$;
4. Associated with each $x \in V$ is a unique element in V, denoted by $-x$, such that $x + (-x) = -x + x = 0$;
5. $(x + y) + z = x + (y + z)$ for all $x, y, z \in X$;
6. $\alpha \cdot x \in V$ for all $x \in V$ and for all $\alpha \in \mathbb{F}$;
7. $\alpha \cdot (x + y) = \alpha \cdot x + \alpha \cdot y$ for all $x, y \in X$ and all $\alpha \in \mathbb{F}$;
8. $(\alpha + \beta) \cdot x = \alpha \cdot x + \beta \cdot x$ for all $x \in V$ and all $\alpha, \beta \in \mathbb{F}$;

9. (αβ) · x = α · (β · x) for all x∈V and all α, β ∈𝔽;
10. 1 · x = x for all x∈V.and 1 ∈ 𝔽.

2.2. Subspace of vector space: - If a subset of vector space is closed under addition and multiplication by scalar. Then it is itself a vector space over the same field.

2.3. Linearly Dependent and independent: - A set of non-zero vectors is linearly dependent if one element of the set can be written as a linear combination of the others. The set is linearly independent if this cannot be done.

2.4. Bases and dimension: - A basis for a vector space is a linearly independent set of vectors such that any vector in the space can be written as a linear combination of elements of this set. The *dimension* of the space is the number of elements in this space

2.5. Norm and normed vector space:- Let X be a vector space. A function$\|\cdot\| : X \longrightarrow [0, \infty[$, which satisfied the following properties is called a norm on X. For all $x, y \in X$,

N1. $\|x\| \geq 0$
N2. $\|x\| = 0$ *if and only if* $x = \mathbf{0}$
N3. $\|\alpha x\| = |\alpha|\|x\|$ *for any scalar* α (Absolute homogeneity)
N4. $\|x + y\| \leq \|x\| + \|y\|$ (Subadditivity).

If $\|\cdot\|$ is a norm on X, then we say that the pair$(X, \|.\|)$, is called a normed vector space or simply normed space.

2.6. Aunu Numbers: - There are two types of Aunu Numbers; the (123)-avoiding class obtained from a recursion relation (Ibrahim and Audu, 2005a) as follows:

$$N(A_n(123)) = \frac{P_n - 1}{2}$$

Give rise to: 2, 3, 5, 6, 8, 9, 11, 14 …
Corresponding to the length of 5, 7,11,13,17, 19 …
The sequence 2,3,5,6,9,11,14… is called the Aunu numbers correspond to the (123)-avoiding class of permutation.
On the other hand the (132)-avoiding class of Aunu permutation patterns is obtained from a relation as follows:

$$N\left(A_n(132)\right) = n + (m - 1),\ m \leq n. \text{ and } n \in \square \geq 3$$

Give rise to: 5,7,9,11,13, …
Corresponding to the length of 3,4,5,6, 7, …
The sequence 5,7,9,11,13… is called the Aunu numbers corresponding to the (132)-avoiding class of permutation
Where $N(A_n(123))$ is the number of the class of numbers expressed as permutations, that avoid (123) patterns while P_n is the n^{th} prime number n ≥ 5.

III. Application in Aunu Permutation Pattern

In this section we will establish easier method of constructing of normed linear apace using a class of (123)-avoiding permutation patterns here under referred to as AUNU pattern; consider the five elements $\{a_1, a_4, a_2, a_0, a_3\}ofZ_5$ Ibrahim (2005b).

Proposition 3.1
Let Ω= $\{a_1, a_4, a_2, a_0, a_3\} \in \Gamma_{Z_5}$ be a cycle of AUNU group of order p, were $p \geq 5$ under binary operation "$*$" on Ω defined by $x * y$ and $x * y$ where $x, y \in \Omega$, "$*$" are arbitrary module 5 respectively. Then Ω is a vector space over finite field Z_p for $2 \leq p \leq 5$ under $*$.
Proof
For a Ω to be a vector space it must satisfy properties of vector space that is
for any $x, y \in V$. The following axioms of vector space hold;
. 1. x + y ∈V whenever x, y ∈ Ω;
2. x+ y = y + x for all x, y ∈ Ω;
3. There exists a unique element in Ω, denoted by 0, such that x + 0 = 0 + x = x
for all x∈ Ω;
4. Associated with each x∈ Ω is a unique element in Ω, denoted by −x, such that x + (−x) = −x + x = 0;
5. (x+ y) + z = x + (y + z) for all x, y, z∈ Ω;
6. α · x∈V for all x∈ Ω and for all α ∈F;
7. α · (x+ y) = α · x + α · y for all x, y ∈ Ω and all α ∈F;
8. (α + β) · x = α · x + β · x for all x∈ Ω and all α, β ∈F;

9. (αβ) · x = α · (β · x) for all x∈ Ω and all α, β ∈F;
10. 1 · x = x for all x∈ Ω. and 1 ∈ 𝔽.
Then, regarding $\{a_1, a_4, a_2, a_0, a_3\}$ as an element to construct a table satisfies properties of vector space.

Table 3.1: operation table of first cycle

*	a_1	a_4	a_2	a_0	a_3
a_1	a_2	a_0	a_3	a_1	a_4
a_4	a_0	a_3	a_1	a_4	a_2
a_2	a_3	a_1	a_4	a_2	a_0
a_0	a_1	a_4	a_2	a_0	a_3
a_3	a_4	a_2	a_0	a_3	a_1

The result follows
For multiplication, let $[a_0, a_1, a_2,]$ be a field F, then

Table 3.2. multiplication

*	$\boldsymbol{a_1}$	$\boldsymbol{a_4}$	$\boldsymbol{a_2}$	$\boldsymbol{a_0}$	$\boldsymbol{a_3}$
$\boldsymbol{a_0}$	$\boldsymbol{a_0}$	$\boldsymbol{a_0}$	$\boldsymbol{a_0}$	$\boldsymbol{a_0}$	$\boldsymbol{a_0}$
$\boldsymbol{a_1}$	$\boldsymbol{a_1}$	$\boldsymbol{a_4}$	$\boldsymbol{a_2}$	$\boldsymbol{a_0}$	$\boldsymbol{a_3}$
$\boldsymbol{a_2}$	$\boldsymbol{a_2}$	$\boldsymbol{a_3}$	$\boldsymbol{a_4}$	$\boldsymbol{a_0}$	$\boldsymbol{a_1}$

Construction of norm using constructed vector Space

A vector space Ω together with a function$\|\cdot\| : \Omega \longrightarrow [0, \infty)$, defined by $\left|\|x\|\right| = |x|$ which is absolute value function defines a norm on Ω. To see this let for all $x, y \in \Omega$,
N1. $\|x\| \geq 0$
N2. $\|x\| = 0 \;\; if\ and\ only\ if\ x = \mathbf{0}$
N3. $\|\alpha x\| = |\alpha x| = |\alpha|\|x\|\ for\ any\ scalar\ \alpha$ (Absolute homogeneity)
N4. $\|x + y\| = \|x\| + \|y\|$ (Subadditivity).
Let β be a fix positive integer and define a function $||\cdot||: \Omega \rightarrow [0, \infty)$ by $\left||x|\right| = \beta x$ then $||\cdot||$ is a norm on Ω.

IV. Conclusion

The application of AUNU permutation pattern on norm space have yield some result. Were the sequence of 123-avoiding pattern was use to construct a vector space and result follows, we also used constructed vector space to construct a norm and normed space was also defined and model of class of (123)-avoiding pattern has been generated.

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