Synchronization of two Chua's Circuits through LC Series Coupling

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Abstract- Synchronization of two chaotic systems by using different coupling schemes like resistance coupling, inductor coupling and capacitor coupling is the most common practice. This can also be achieved by the coupling of two Chua's circuits by means of a LC series combination. The changing of any one of the parameters L or C of series coupling element will introduce the change in the dynamics of the coupled system and the complete in-phase synchronization is observed when these parameters assume certain small values. As the coupling parameter value increases, the anti-phase oscillations begin to grow and for the large values of coupling inductance and coupling capacitance, only the almost complete synchronization of anti-phase oscillations is observed. Since the initial complete synchronization state of in-phase oscillations is not observed even for the large values of coupling inductance and coupling capacitance, no chaotic itinerancy is observed in case of series LC coupled identical Chua's circuits. Apart from this, the intermittent or alternately torus formation and destruction routes are also appeared in the synchronization transition. The coupling parameter values for which the complete synchronization is observed in study can be used for constructing the secure communication systems in the future.

Index Terms- Chaotic bursting, Chaotic itinerancy, Chua's circuit, Complete synchronization, LC series coupling.

I. Introduction

Chaotic systems are very important physical systems in nature with some peculiar properties and potential applications. The Chua's circuit is one such a system with simple structure and rich chaotic characteristics [1,2]. Synchronization of the chaotic systems is an interesting phenomenon because it shows some cooperative behaviour within the chaos of the system. The synchronized chaotic systems are important because such systems can be used for some potential applications of signal processing and secure communications.

It is impossible to apply the same initial conditions for any two isolated systems in the nature. Since any chaotic system sensitively depends on the initial conditions, this limitation leads to an extreme divergence of the paths which are initially very close to each other in the phase space. So, the synchronization or correlation in time evolution is not possible even for any two identical chaotic systems which are independent of each other[3].

The simplest way of synchronization of any two chaotic systems is the linear coupling[4]. This coupling facilitates the more energy or current exchange between the two chaotic circuits. Furthermore, this linear coupling scheme can be used for the synchronization of some specific chaotic systems like the Chua's circuit.

The large number of experimental and numerical studies of synchronization of the chaotic systems were carried out for the Chua's circuit in Kennedy's implementation[5,6,7].Chua et al. studied the problem of synchronization of two Chua's circuits by means of linear coupling with a resistor and it was found that the synchronization can be achieved for different values of coupling resistance in different coupling schemes[8].

Synchronization studies with different coupling elements like capacitors and inductors are also carried out with two Chua's circuits. Astakov et al. carried out some numerical and experimental studies to study the non-linear dynamics of the two identical Chua's circuits in capacitive coupling[9].They observed the coupled system to have a few new modes of oscillations compared to that of the resistive coupling. Yu-meng XU et al. also carried out the numerical studies for two Pikovsk-Rabinovich non-linear circuits with same coupling element and they found that for appropriate electric field strength across the coupling capacitor the synchronization can be achieved[10]. Liu Zhilong et al. also carried out the study of two Chua's circuits in capacitive coupling and observed that the complete synchronization can be achieved for the proper electric field intensity across the coupling capacitor[11].

Zhao Yao et al. are carried out the numerical studies of two Chua's circuits in inductor coupling [12]. In this study, they observed that the complete synchronization can be achieved for a suitable intensity of the time varying magnetic field across the coupling inductor.

In the present paper, a series LC circuit is used as a coupling element between the two identical Chua's circuits and the synchronization problem is studied with the **LTspice** software[13]. The phase space dynamics of two identical Chua's circuits is observed for different values of inductance or capacitance of LC series coupling element. The dynamics of the coupled Chua's circuits is observed by observing the evolution of phase space portrait from an initial straight line to a final straight line superimposed by some non-vanishing small amplitude transverse oscillations. The dynamics of the LC series coupled system is observed to be somewhat different from that of the individual Chua's circuit with the presence of some new modes of oscillations in the corresponding phase space portrait. The straight line in phase portrait making 45° angle with x-axis always shows the complete synchronization of two identical Chua's circuits. This means that the output values of two identical Chua's circuits share a common point in the phase space with respect to the time. As the coupling parameter value increases, some intermittent small amplitude oscillations around a stable in-phase synchronized state are observed. So the complete synchronization of in-phase oscillations is only possible for the appropriate small values of coupling inductance and coupling capacitance of the LC series coupling element. At the higher values of coupling parameters only an almost synchronized state of anti-phase oscillations is observed. So there is no chaotic itinerancy in the chaotic evolution of the coupled system. Apart from this, the synchronization transition witnessed the intermittent, torus formation and destruction routes of chaotic evolution with the relatively more complicated phase portraits compared to that of the simple coupling elements.

II. MODEL AND SCHEME

Chua's circuit consists of - one inductor, two capacitors, one linear resistor R and one nonlinear resistor N_R , as shown in Fig1.The two Chua's circuits are realized in Kennedy's implementation where the non-linear resistor N_R is made up of two op-amps and six resistors, as shown in Fig2. This provides the very inexpensive and the robust realization of the Chua's circuit. The values of all the parameters in the Chua's circuits are fixed as shown in the Kennedy's paper(1992). In this paper, two such identical Chua's circuits are coupled with a series LC combination as shown in Fig3. All the circuits used in the paper are constructed by using *LTspice* software. All the procedures to install this software and draw wires, including the components and run simulations can be found in *LTspiceGettingStartedGuide*[14].

The dynamics of coupled Chua's circuits is studied by running the simulations for different values of coupling inductance(L_3) and coupling capacitance(C_5) and then observing the phase space portraits between voltages $V_{C_4}$and V_{C_2} of uncoupled capacitors on the plot pane of the LTspice schematic editor.

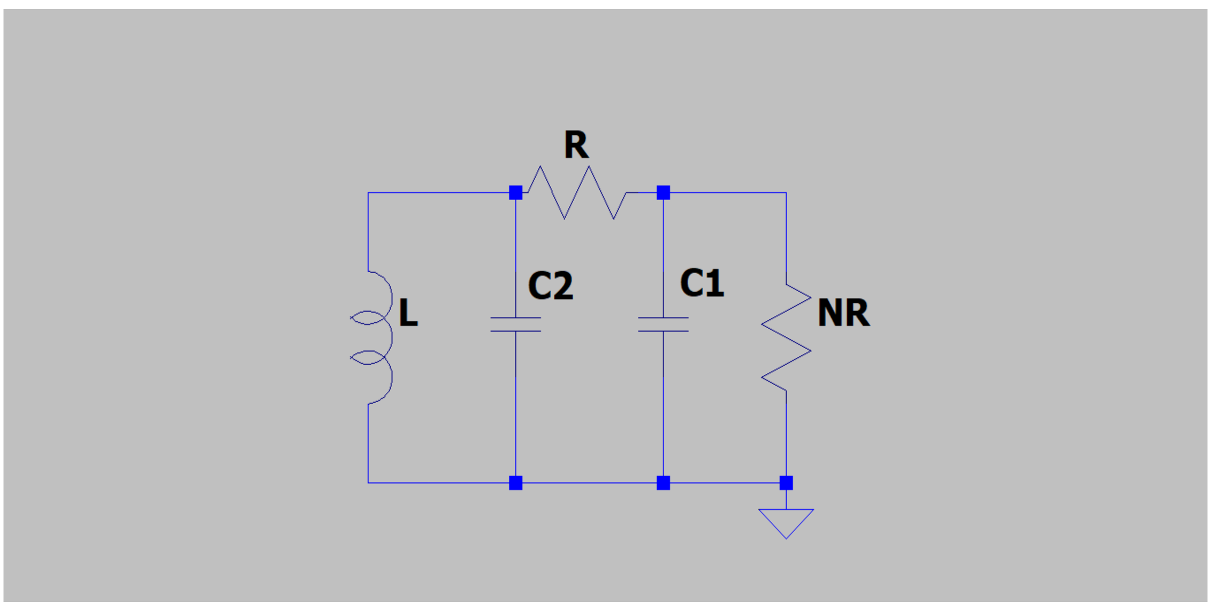


Fig.1.Schematic diagram of Chua's Circuit drawn with LTspice software

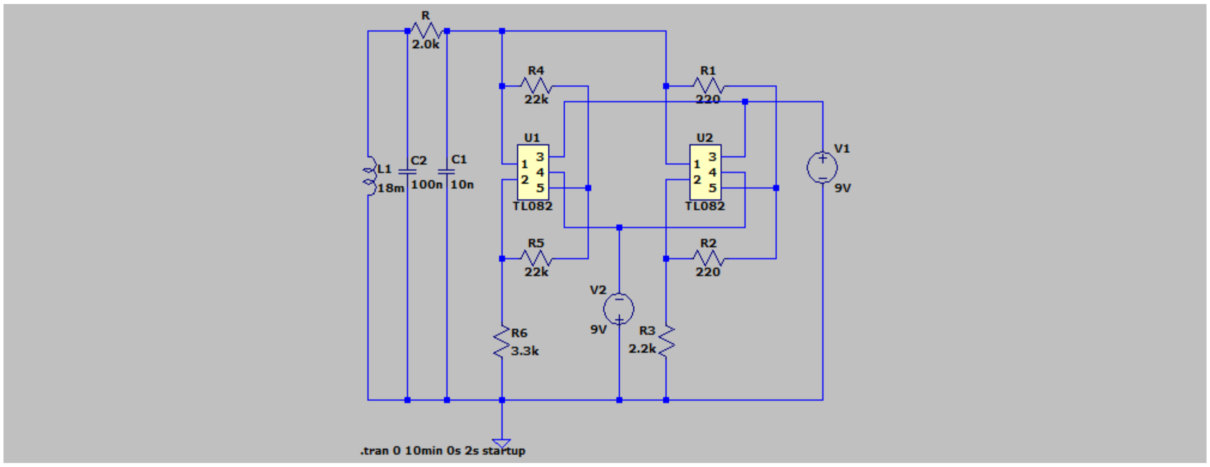


Fig.2.Chua's circuit realization (in Kennedy's implementation) on the Schematic Editor of LTspice

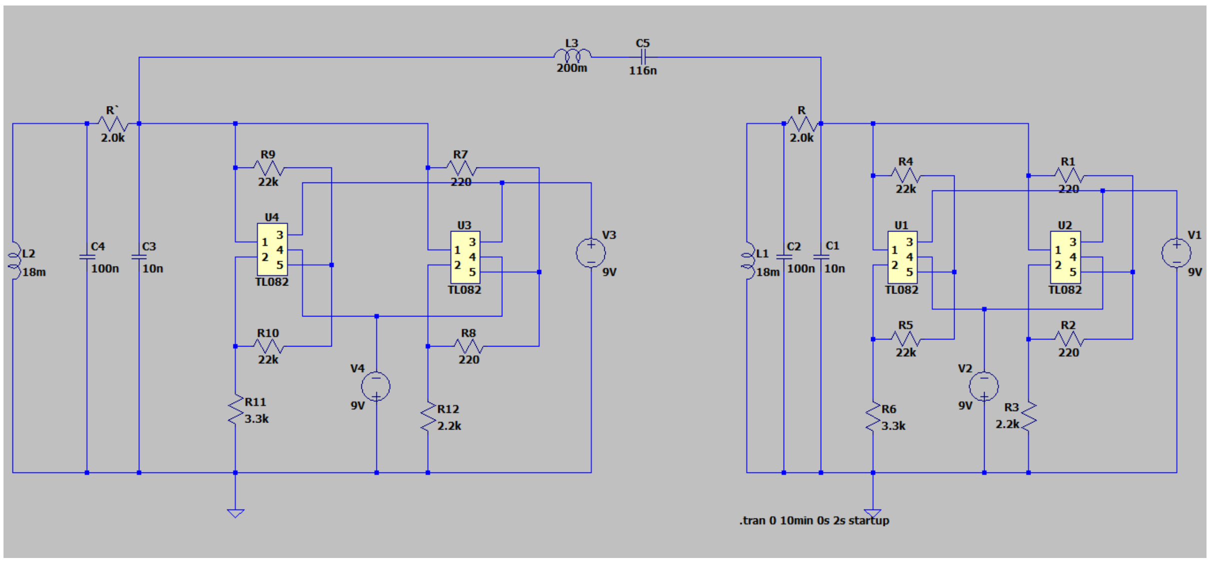


Fig.3.LC series coupling of two Chua's circuits on Schematic Editor of LTspice

The dynamics of the coupled system is characterised by the following set of equations obtained by applying the Kirchhoff's laws at the two end points of the linear resistor R of a Chua's circuit.

Applying Kirchhoff's current laws at the two end points of the resistor R:

$$C_2\left(\frac{dV_{C_2}}{dt}\right)=I_{L_1}+\left(\frac{V_{C_1}-V_{C_2}}{R}\right) \quad (1)$$

where V_{C_1} is the voltage across the capacitor with capacitance C_1

V_{C_2} is the voltage across the capacitor with capacitance C_2

and I_{L_1} is the current flowing through the inductor with inductance L_1

Similarly:

$$\left(\frac{V_{C_1}-V_{C_2}}{R}\right)=C_2\left(\frac{dV_{C_2}}{dt}\right)+f\,(V_{C_1})+I \quad (2)$$

where f $(V_{C_1})$is the function giving characteristics of the Chua's diode N_R

and I is the current flowing through the series coupling element

Here piecewise-linear function f (V_{C_1}) is defined as follows:

$$f\,(V_{C_1})=G_bV_{C_1}+0.5(G_a\text{-}G_b)\,(|V_{C_1}+E|-|V_{C_1}-E|) \quad (3)$$

where G_a,G_b are the conductance values

and E is the breaking point voltage

Noting that voltage developed between the two ends of the capacitor with capacitance C_2 is equal to that of the inductor with inductance L_1 :

$$L_1 \frac{dI_{L_1}}{dt} = -V_{C_2} \quad (4)$$

where I_{L_1} is the current flowing through the inductor with inductance L_1

Similarly, due to the series coupling between C_1 and C_3:

$$V_{L_3} + V_{C_5} = V_{C_1} - V_{C_3} \quad (5)$$

Since the same current flows through the series: $I_{L_3} = I_{c_5} = I$:

$$I(X_{L_3} - X_{C_5}) = V_{C_1} - V_{C_3} \quad (6)$$

where X_{L_3}(=ωL_3) and X_{C_5}(=1/ωC_5) are the *reactances* of coupling inductor and coupling capacitor respectively and $(X_{L_3} - X_{C_5})$ is the impedance of the series LC circuit.

Applying the scale transformation of the variables, the dynamical equations in dimensionless form are as follows:

Using Eq(6):

$$\dot{x} = \alpha[y - x - f(x) + \eta(x - x')] \quad (7)$$

$$\dot{y} = x - y + z \quad (8)$$

$$\dot{z} = -\beta y \quad (9)$$

where $\alpha = \frac{C_2}{C_1}$, $\beta = \frac{C_2 R^2}{L_1}$, $\eta = \left(\frac{R}{X_{L_3} - X_{C_5}}\right)$ (10)

$\mathbf{x} = V_{C_1}/E$, $x' = V_{C_3}/E$, $\mathbf{y} = V_{C_2}/E$, $\mathbf{z} = I_{L_1} R$ /E

τ = t/$R\ C_2$, $X_{L_3} = \omega L_3$, $X_{C_5} = 1/\omega C_5$ and $\dot{x} = \left(\frac{dx}{d\tau}\right)$ etc.

Another set of equations are given by applying the Kirchhoff laws at the resistor R':

$$\dot{x}' = \alpha'[y' - x' - f(x') - \eta'(x - x')] \quad (11)$$

$$\dot{y}' = x' - y' + z' \quad (12)$$

$$\dot{z}' = -\beta' y' \quad (13)$$

where $\alpha' = \frac{C_4}{C_3}$, $\beta' = \frac{C_4 {R'}^2}{L_2}$, $\eta' = \left(\frac{R'}{X_{L_3} - X_{C_5}}\right)$ (14)

$x = \frac{V_{C_1}}{E}$, $x' = V_{C_3}/E$, $y' = V_{C_4}/E$, $z' = I_{L_2} R'$/E

τ = t/$R' C_4$, $X_{L_3} = \omega L_3$, $X_{C_5} = 1/\omega C_5$ and $\dot{x}' = \left(\frac{dx'}{d\tau}\right)$ etc.

Here the two Chua's circuits are considered to be identical and hence, $\alpha = \alpha'$, $\beta = \beta'$, $\eta = \eta'$, $R_1 = R_2 = R_7 = R_8$ = 220 Ω, $R_3 = R_{12}$ = 2.2 kΩ, $R_4 = R_5 = R_9 = R_{10}$ = 22 kΩ, $R_6 = R_{11}$ = 3.3 kΩ, $C_1 = C_3$ = 10 nF , $C_2 = C_4$ =100 nF, $L_1 = L_2$ =18 mH as shown in the Kennedy's paper(1992). The resistance which is a variable parameter in each individual Chua's circuit is now fixed as R = R′ = 2.0 kΩ.

Transforming m_0=RG_a and m_1=RG_b , the piecewise linear function can also be expressed now in the following dimensionless form:

$$f(x) = m_1 x + 0.5(m_0 - m_1)(|x+1| - |x-1|) \quad (15)$$

$$f(x') = m_1 x' + 0.5(m_0 - m_1)(|x'+1| - |x'-1|) \quad (16)$$

Here x and x' are defied as x= V_{C_1}/E and $x' = V_{C_3}/E$ respectively.

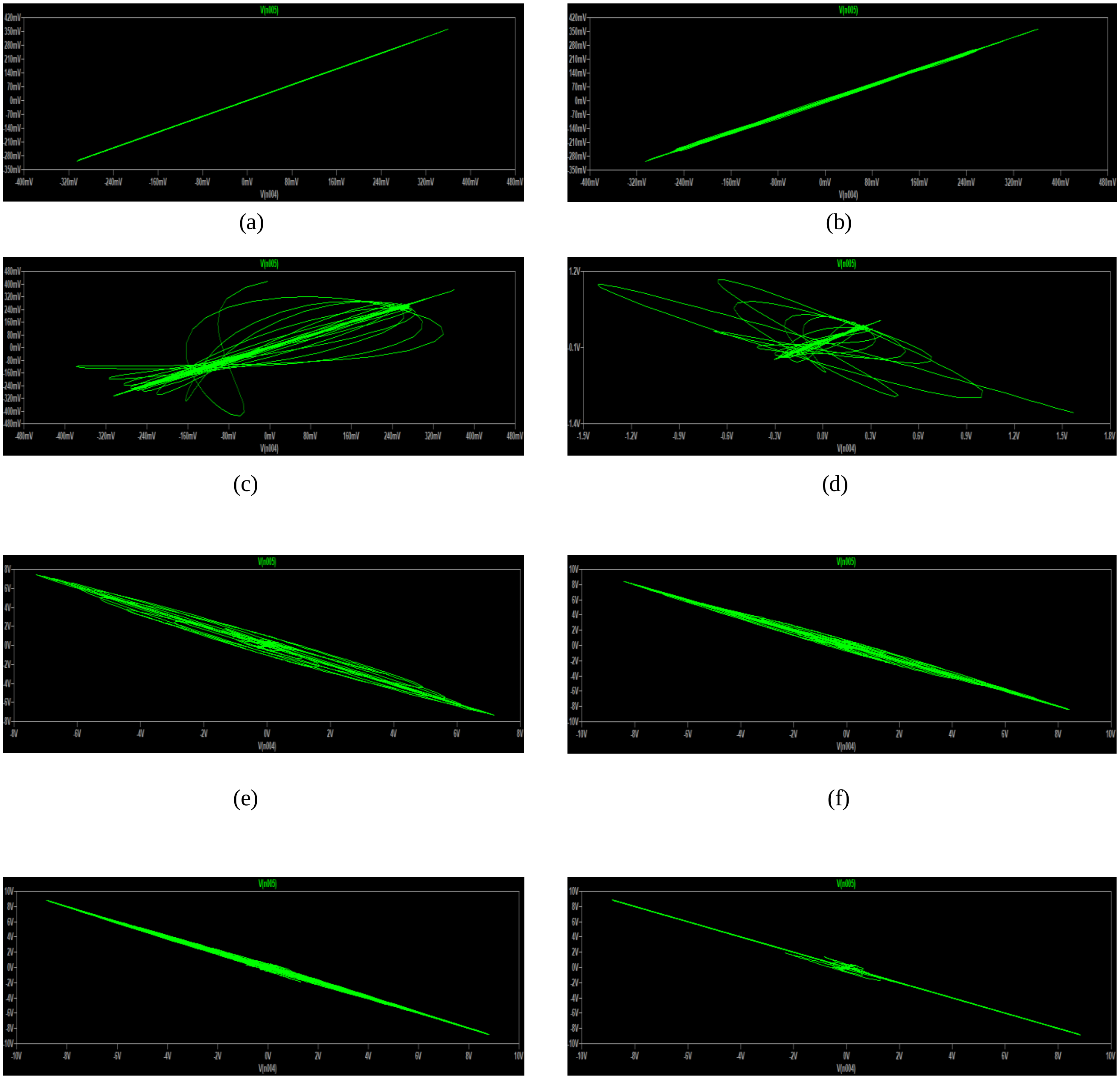
So, from Eq(1) to Eq(16) describe the synchronization phenomenon in LC series coupled Chua's circuits. For some particular coupling values of coupling parameters L_3 and C_5, the complete synchronization is possible such that $(x - x') \to 0$ as the time → ∞ .

III. RESULTS AND DISCUSSION

At the beginning of the simulations, irrespective of the values of either coupling inductance L_3 or coupling capacitance C_5, the phase portrait is observed to be momentarily a straight line corresponding to the in-phase synchronization of the oscillations of two identical Chua's circuits. But later on, it appears to be a random trace on the phase space due to the chaotic nature of the two Chua's circuits. So, for the synchronization of the outputs of the two identical Chua's circuits, the variation of values of L_3 or C_5 is needed in the LT spice simulations. The chaotic dynamics of the coupled system is observed near the complete synchronization state in order to observe the chaos at the synchronization transition.

A. Changing of coupling inductance:

Coupling inductance is varied at a fixed coupling capacitance value of C_5= 10nF. With the gradual increase of the coupling inductance, the complete synchronization of in-phase oscillations is observed initially at L_3= 260mH, as shown in Fig.4(a). For the further increase of coupling inductance L_3 value, the synchronization is observed to be lost completely at the intermediate value L_3=290mH. Here the system reaches to a bursting chaotic state as shown in Fig.4(d), through the states shown in Fig.4(b) and Fig.4(c). This state is a transitory state switching in between the in-phase and anti-phase oscillations. After this stage the anti-phase oscillations begin to grow with the disappearance of in-phase oscillations as shown in Fig.4(e), Fig.4(f) and Fig.4(g). For the further increasing of coupling inductance value, the almost complete synchronization of anti-phase oscillations takes place at L_3 = 342mH, as shown in Fig.4(h). After reaching of this value, the complete synchronization of neither anti-phase nor in-phase oscillations is observed even for the higher values of coupling inductance. So, there is no chaotic itinerancy in case of variation of coupling inductance. Furthermore, this chaotic evolution can be identified with torus formation or destruction or alternately intermittent route to chaos but with some complex manner compared to that of other simple coupling elements.



(a) (b) (c) (d) (e) (f) (g) (h)

Fig.4.Phase portraits at different values of L_3 from (low to high): (a) At 260 mH (b) At 270 mH (c) At 280 mH (d) At 290mH (e) At 300mH (f) At 320 mH (g) At 340 mH (h) At 342 mH (X-axis:V_{C_4} and Y-axis:V_{C_2})

At the small coupling inductance values, initially the synchronization of in-phase oscillations is observed because of small reactance value of the inductor. With the increase of coupling inductance value, the induced emf across the coupling inductor will modulate the output signals of both Chua's circuits so that the synchronization of anti-phase oscillations is observed. This happens to a large extent but the complete synchronization is not observed for any large value of coupling inductance due to the affect of large series reactance of the inductor.

B. Changing of coupling capacitance:

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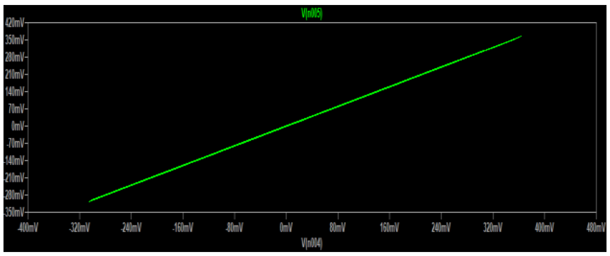
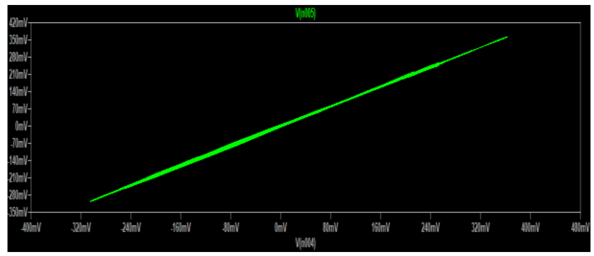
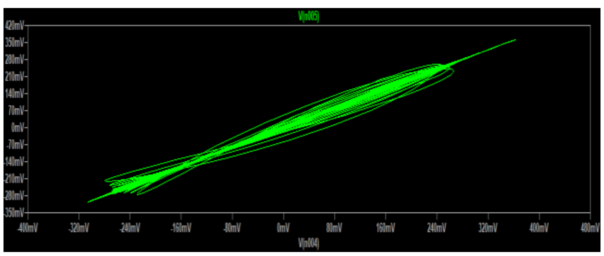
The synchronization transition is observed with variation of coupling capacitance C_5 at the fixed coupling inductance value of L_3 = 200 mH. As observed in the case of coupling inductance L_3 variation, the complete synchronization of in-phase oscillations is observed at a low value coupling capacitance C_5 = 20nF, as shown in Fig.5(a). As the coupling capacitance increases, initially few small amplitude oscillations around the stable line will emerge intermittently as shown in Fig.5(b), Fig.5(c), Fig.5(d) and Fig.5(e) and finally this leads the system to a chaotic bursting state at C_5 =25nF, as shown in Fig.5(f). This happens in the case of changing of coupling inductance L_3 value also. After this stage the transition from in-phase to anti-phase oscillations takes place through states shown in Fig.5(g) and Fig.5(h). Finally at C_5= 28nF, an almost complete synchronization state with few small amplitude oscillations still about a stable line of anti-phase oscillations appears, as shown in Fig.5(i). After C_5 is reached to this value, the perfect complete synchronization is not observed even for the next higher values of it. So, there is no chaotic itinerancy with the varying of coupling capacitance also. Apart from this, the chaotic evolution of the system is observed to follow the torus formation and break down or alternately intermittent routes to chaos, as in the case of inductance variation.



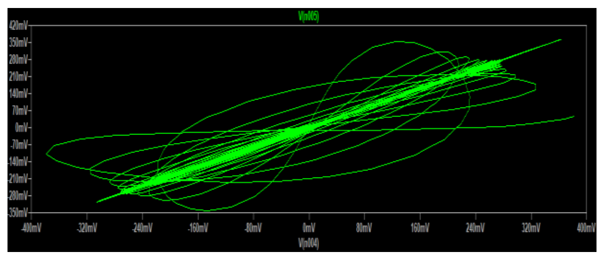
(a)



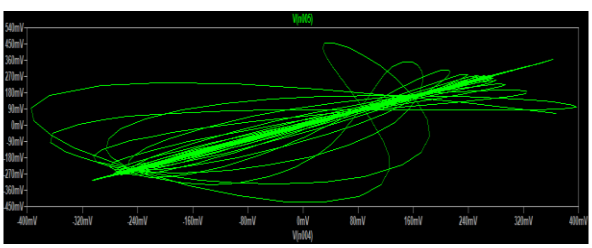
(b)



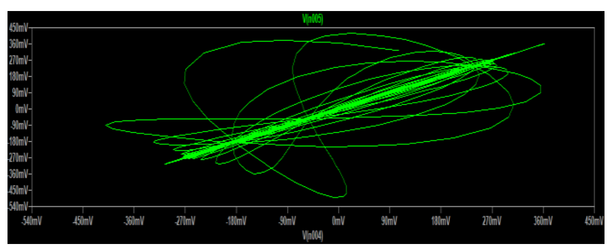
(c)



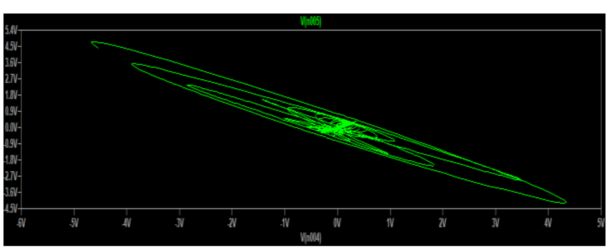
(d)



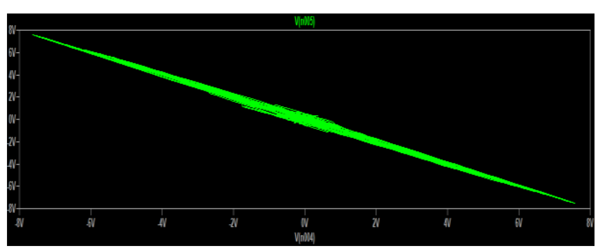
(e)



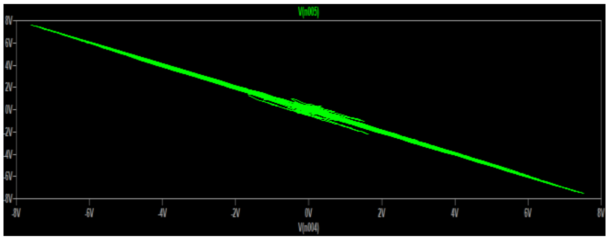
(f)



(g)



(h)



(i)

Fig.5. Phase portraits at different values of C_5(low to high):(a) At 20 nF (b) At 21 nF (c) At 22 nF (d) At 23 nF (e) At 24 nF (f) At 25 nF (g) At 26 nF (h) At 27 nF (i) At 28 nF(X-axis:V_{C_4} and Y-axis:V_{C_2})

At the small coupling capacitance values, the complete synchronization of in-phase oscillations is observed due to the affect of series inductor. For the further increase of coupling capacitance value, the increased electric field across the capacitor will change the outputs of the individual Chua's circuits and hence the anti-phase oscillations begin to grow with the disappearance of in-phase oscillations. Even at the high coupling capacitance values, the only an almost synchronization of anti-phase oscillations is observed because of the series coupling inductance and its large induced back emf.

IV. CONCLUSIONS

When the two identical Chua's circuits are coupled through a LC series circuit, the change in coupling inductance or coupling capacitance from low to high values will leads to the transition from complete synchronization of in-phase oscillations to almost synchronized state of anti-phase oscillations through the bursting chaotic state. The reason for this transition is that the phases of the outputs of the identical Chua's circuits relatively change from one phase to other due to the change in the relative phase of the current passing through the LC circuit for the variation of the reactance of the coupling inductance and capacitance. So, the complete synchronization of in phase oscillations is only observed at the small values of coupling inductance and coupling capacitance .At the large values of these coupling parameters, only an almost complete synchronization state of anti-phase oscillations is observed due to reactance of the series circuit elements. Since the coupled system is not observed again in the same state of complete synchronization of in-phase oscillations even for the higher values of the coupling parameters, no chaotic itinerancy is observed in LC series coupled Chua's circuits. Apart from this, the intermittent route or alternately, the torus formation and torus destruction routes are also appeared in the synchronization transition. Furthermore, the set of parameter values for which the complete synchronization is predicted in the study can be used for constructing the secure communication systems in the future.

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