On Curvature Collineation And Conformal Motion In A Finsler Space

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Abstract: The present communication respectively deals with projective curvature collineation, projective conformal motion, H-curvature collineation and H –conformal motion .

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1.INTRODUCTION

Berwald introduced a connection coefficient $G^i_{jk}(x,\dot{x})$ defined by

$$G^i_{jk}(x,\dot{x}) \overset{def}{=} \frac{\partial^2 G^i}{\partial \dot{x}^j \partial \dot{x}^k} \tag{1.1}$$

and accordingly the covariant derivative of an arbitrary contravariant vector X^i is given by Rund [6]

$$X^i_{(j)} = \frac{\partial X^i}{\partial x^j} - \frac{\partial X^i}{\partial \dot{x}^h}\frac{\partial G^h}{\partial \dot{x}^j} + G^i_{jh}X^h \tag{1.2}$$

The function G^i appearing in (1.2) are positively homogeneous of degree two in its directional arguments $\dot{x}^i$ and satisfy the following identities :

$$\text{(a)} \quad G^i{}_{hkr}\dot{x}^r = G^i_{hkr}\dot{x}^k = G^i_{hkr}\dot{x}^h = 0, \tag{1.3}$$

$$\text{(b)} \; G^i{}_{hk}\dot{x}^h = 0 \quad \text{(c)} \quad G^i{}_k\dot{x}^k = 2G^i.$$

The geodesic deviation has been given by Rund[6] in the following form

$$\frac{\partial^2 Z^j}{\partial u^2} + H^j_k(x,\dot{x})Z^k = 0 \tag{1.4}$$

where ,the vector Z^i is called the variation vector and the tensor $H^j_k(x,\dot{x})$ is called deviation tensor being defined by

$$H^i_k = 2\partial_k G^i - \partial_h \dot{\partial}_k G^i \dot{x}^h + 2G^i_{kl} G^l - \dot{\partial}_l G^i \dot{\partial}_k G^l \qquad (1.5)$$

the tensors defined by

(a) $$H^i_{jk}(x,\dot{x}) \overset{def}{=} \frac{1}{3}\left(\frac{\partial H^i_k}{\partial \dot{x}^j} - \frac{\partial H^i_j}{\partial \dot{x}^k}\right) \qquad (1.6)$$

(b) $$H^i_{jkl}(x,\dot{x}) = \frac{\partial H^i_{jl}}{\partial \dot{x}^k}.$$

These tensors can alternatively be expressed in terms of $G^i(x,\dot{x})$ in the following alternative forms

(a) $$H^i_{jk}(x,\dot{x}) = \frac{\partial^2 G^i}{\partial x^k \partial \dot{x}^j} - \frac{\partial^2 G^i}{\partial x^j \partial \dot{x}^k} + G^i_{kr}\frac{\partial G^r}{\partial \dot{x}^j} - G^i_{rj}\frac{\partial G^r}{\partial \dot{x}^k} \qquad (1.7)$$

(b) $$H^i_{hjk}(x,\dot{x}) = \frac{\partial G^i_{hj}}{\partial x^k} - \frac{\partial G^i_{hk}}{\partial x^j} + G^r_{hj}G^i_{rk} - G^r_{hk}G^i_{rj} +$$

$$+G^i_{rhk}\frac{\partial G^r}{\partial x^j} - G^i_{rhj}\frac{\partial G^r}{\partial \dot{x}^k}$$

where

(a) $$G^i_{hjk} \overset{def}{=} \frac{\partial G^i_{hj}}{\partial \dot{x}^k} \qquad (1.8)$$

and (b) $$G^i_{hjk}\dot{x}^k = 0.$$

The tensors $H^i_{jk}(x,\dot{x})$ and $H^i_{jkl}(x,\dot{x})$ are respectively named as Berwald's deviation tensor and Berwald's curvature tensor .We have the following commutation formula [7] involving the Lie-derivative and Berwald's covariant derivative

(a) $$\dot{\partial}_l(\pounds_v T^i_j) - \pounds_v(\dot{\partial}_l T^i_j) = 0\ , \qquad (1.9)$$

(b) $$\pounds_v T^i_{j(k)} - (\pounds_v T^i_j)_k = T^h_j \pounds_v G^i_{kh} - T^i_h \pounds_v G^h_{kj} - (\dot{\partial} T^i_j)\pounds_v G^h_{ks}\dot{x}^s$$

and (c) $$(\pounds_v G^i_{hk})_{(j)} - (\pounds_v G^i_{jh})_{(k)} = \pounds_v H^i_{hkj} + (\pounds_v G^l_{jb})\dot{x}^b G^i_{hkl} - (\pounds_v G^l_{kb})\dot{x}^b G^i_{hjl}.$$

The commutation formula involving the tensors H^i_{jkl} and G^i_{jkl} are respectively given by

(a) $$T^i_{j(h)(k)} - T^i_{j(k)(h)} = -\dot{\partial}_l T^i_j H^r_{hk} - T^i_r H^r_{jkh} + T^r_j H^i_{rhk} \qquad (1.10)$$

(b) $$(\dot{\partial}_k T^i_j)_l - \dot{\partial}_k(T^i_{j(l)}) = T^i_r G^r_{jkh} - T^r_j G^i_{rkh}.$$

The Berwald's curvature tensor also satisfies the following identities and contractions

(a) $H_i \dot{x}^i = (n-1)H$, (b) $H_{jh} - H_{hj} = H^k_{khj}$ (1.11)

(c) $H^j_{ki}\dot{x}^k = H^j_i = H^j_{ik}\dot{x}^k$

and (d) $H^h_{hjk} - H^k_{jhk} = \partial_h G^k_{jk} - \partial_j G^k_{hk} + G^r_{jk}G^k_{rh} - G^r_{hk}G^k_{rj} + \partial_j G^r G^k_{hrk} - \partial_h G^r G^k_{jrk}$.

The projective covariant derivative of an arbitrary tensor field $T^i_j(x,\dot{x})$ with respect to x^k is given by [5] as

$$T^i_{j((k))} = \partial_k T^i_j - \dot{\partial}_s T^i_j \Pi^s_{rk}\dot{x}^r + T^h_j \Pi^i_{hk} - T^i_h \Pi^h_{jk} \tag{1.12}$$

where ,

$$\Pi^i_{jk}(x,\dot{x}) \overset{def}{=} \{G^i_{jk} - \frac{1}{n+1}(2\delta^i_{(j}G^r_{<r>k)} + \dot{x}^i G^r_{rkj})\} \tag{1.13}$$

is called projective connection coefficient and these coefficients are symmetric in its lower indices .

Involving the projective covariant derivative ,we have the following commutation formulae [5]

(a) $\partial_h (T^i_{j((k))}) - (\partial_h T^i_j)_{((k))} = T^s_j \Pi^i_{shk} - T^i_s \Pi^s_{jhk}$ and (1.14)

(b) $2T^i_{j[((h))((k))]} = -\dot{\partial}_r T^i_j Q^r_{shk}\dot{x}^s + T^s_j Q^i_{shk} - T^i_s Q^s_{jhk}$

where ,

$$Q^i_{hjk}\dot{x}^s \overset{def}{=} 2\{\partial_{[k}\Pi^i_{j]h} - \Pi^i_{rh[j}\Pi^r_{k]} + \Pi^r_{h[j}\Pi^i_{k]r} \tag{1.15}$$

is called the projective entity and satisfies the following relation

(a) $Q^i_{hjk} + Q^i_{jkh} + Q^i_{khj} = 0$, (1.16)

(b) $Q^i_{hjk((s))} + Q^i_{hks((j))} + Q^i_{hsj((k))} = 0$

(c) $Q^i_{hjk} = -Q^i_{hkj}$, (d) $Q^i_{jk} = \frac{2}{3}\dot{\partial}_{[j}Q^i_{k]}$,

(e) $Q^i_{hjk} = \dot{\partial}_h Q^i_{jk}$, (f) $Q^i_{ijk} = Q_{jk}$, (g) $Q^i_k \dot{x}^k = 0$

(h) $Q^i_{jk} = -Q^i_{kj}$ and (i) $Q^i_{hk}\dot{x}^h = Q^i_k$.

The projective connection coefficients $\Pi^i_{jk}(x,\dot{x})$ satisfy the following relations :

(a) $\Pi^i_{hkr} = \dot{\partial}_h \Pi^i_{kr}$ (b) $\Pi^i_{hk} = \dot{\partial}_h \Pi^i_k$ (1.17)

(c) $\Pi^i_{hkr}\dot{x}^h = 0$ and (d) $\Pi^i_{hk}\dot{x}^h = \Pi^i_k$.

2.PROJECTIVE CURVATURE COLLINEATION

In view of the projective covariant derivative as has been given by (1.12) and the projective connection coefficient $\Pi^i_{jk}(x,\dot{x})$ as has been given by (1.13) ,the Lie-derivative of an arbitrary tensor $T^i_j(x,\dot{x})$ and the projective connection coefficient $\Pi^i_{jk}(x,\dot{x})$ are respectively given by

$$\pounds_v T^i_j(x,\dot{x}) = T^i_{j((r))}v^r + (\dot{\partial}_s T^i_j)v^s_{((r))}\dot{x}^r - T^r_j v^i_{((r))} + T^i_r v^r_{((j))} \tag{2.1}$$

and

$$\pounds_v \Pi^i_{mk}(x,\dot{x}) = v^i_{((m))((k))} + Q^i_{mkr}v^r + (\dot{\partial}_r \Pi^i_{mk})v^r_{((s))}\dot{x}^s \tag{2.2}$$

where , Q^i_{mkr} defined by (1.15).

In between the operators $\pounds_v$, $\dot{\partial}$ and (( )) ,we have the following commutation formulae

$$\text{(a)} \quad \dot{\partial}_\rho(\pounds_v T^i_j) - \pounds_v(\dot{\partial}_\rho T^i_j) = 0, \tag{2.3}$$

$$\text{(b)} \quad (\pounds_v T^i_j)_{((r))} - \pounds_v T^i_{j((r))} = T^l_j \pounds_v \Pi^i_{lr} - T^i_l \pounds_v \Pi^l_{rj} - (\dot{\partial}_l T^i_j)\pounds_v \Pi^l_{rm}\dot{x}^m$$

and (c) $$(\pounds_v \Pi^i_{hj})_{((k))} - (\pounds_v \Pi^i_{hk})_{((j))} = \pounds_v Q^i_{hkj} + (\pounds_v \Pi^l_{jb})\dot{x}^b \Pi^i_{hkl} - (\pounds_v \Pi^l_{kb})\dot{x}^b \Pi^i_{jhl} .$$

We now consider an infinitesimal point transformation given by

$$\bar{x}^i = x^i + v^i(x)dt \tag{2.4}$$

where , $v^i(x)$ stands for a non-zero contravariant vector field defined over the domain of the space and dt is an infinitesimal constant ,such an infinitesimal point transformation defines a projective curvature collineation provided the space F_n admits a vector field $v^i(x)$ such that

$$\pounds_v Q^i_{jkh} = 0 \tag{2.5}$$

where as ,the infinitesimal point transformation under consideration defines a Ricci projective curvature collineation provided there exists a vector field $v^i(x)$ satisfying

$$\pounds_v Q_{hk} = 0 \quad . \tag{2.6}$$

The necessary and sufficient condition in order that the infinitesimal point transformation (2.4) be a projective motion in an F_n is given by

$$\pounds_v \Pi^i_{jk}(x,\dot{x}) = \delta^i_j \in_k + \delta^i_k \in_j \tag{2.7}$$

where, $\Pi^i_{jk}(x,\dot{x})$ is the projective connection coefficient as has been given in (1.13) and $\in_k$ is an arbitrary non-zero covariant vector field satisfying

$$\in_k \dot{x}^k = 0 \quad . \tag{2.8}$$

We now propose to investigate the conditions under which a special projective motion will become a projective curvature collineation .

Keeping in mind (2.3 c) and (2.7) ,the Lie-derivative of the projective entity $Q^i_{hjk}(x,\dot{x})$ can be written in the following form

$$\begin{aligned}\pounds_v Q^i_{hjk} &= \delta^i_h \in_{j((k))} + \delta^i_j \in_{h((k))} - \delta^i_k \in_{h((j))} - \delta^i_h \in_{h((j))} - \\ &\quad - \delta^r_l \in_k \Pi^i_{rjh}\dot{x}^l - \delta^r_k \in_l \Pi^i_{rjh}\dot{x}^l + \delta^r_l \in_j \Pi^i_{rhk}\dot{x}^l + \delta^r_j \in_l \Pi^i_{rhk}\dot{x}^l \quad .\end{aligned} \tag{2.9}$$

In view of (1.15) ,(2.8) and the fact that $\in_k = \partial_k \in$,we get from (2.9) the following

$$\pounds_v Q^i_{hjk} = \delta^i_j \in_{h((k))} - \delta^i_k \in_{h((j))} \tag{2.10}$$

at this stage ,if we assume that the special projective motion becomes a projective curvature collineation then from (2.5) and (2.10) ,we get

$$\delta^i_j \in_{h((k))} - \delta^i_k \in_{h((j))} = 0 \quad . \tag{2.11}$$

We now allow a contraction in (2.11) with respect to the indices i and j and get

$$(n-1) \in_{h((k))} = 0 \tag{2.12}$$

in the light of (2.12) ,we can therefore state :

THEOREM(2.1):

In a Finsler space F_n ,the arbitrary covariant vector field $\in_h$ behaves like a projective covariant constant provided the special projective motion becomes a projective curvature collineation .

Contracting (2.10) with respect to the indices i and k and thereafter using (1.16) ,we get

$$\pounds_v Q_{hj}(x,\dot{x}) = (1-n) \in_{h((j))} \quad . \tag{2.13}$$

In view of (2.6) ,the equation (2.13) gives

$$(1-n) \in_{h((j))} = 0 \quad . \tag{2.14}$$

In the light of (2.14) ,we can therefore state :

THEOREM (2.2):

In a Finsler space F_n ,the arbitrary covariant vector field $\in_h$ behaves like a projective covariant constant provided the special projective motion becomes a Ricci-projective curvature collineation .

We now apply the commutation formula (1.14 b) to $Q^i_j(x,\dot{x})$ and thereafter use (2.7) and get

$$(\pounds_v Q^i_j)_{((k))} = Q^i_k \in_j - Q^h_j \delta^i_k \in_h + (\dot{\partial}_k Q^i_j \in_s + \dot{\partial}_s Q^i_j \in_k)\dot{x}^s \quad . \tag{2.15}$$

Contracting (2.15) with respect to the indices i and j ,we get

$$(\pounds_v Q^i_i)_{((k))} = (\dot{\partial}_k Q^i_i \in_s + \dot{\partial}_s Q^i_i \in_k)\dot{x}^s . \tag{2.16}$$

Therefore ,we can state :

THEOREM (2.3):

If a projective Finsler space admits a special projective motion then (2.17) always holds .

In analogy to (1.14) ,we have the following commutation formula

$$\dot{\partial}_h(Q^i_{jk((m))}) - (\dot{\partial}_h Q^i_{jk})_{((m))} = Q^s_{jk}\Pi^i_{hms} - 2Q^i_{s[k}\Pi^s_{j]hm} \quad . \tag{2.17}$$

Using (2.16) in (2.17) along with the facts that in a projective symmetric Finsler space ,we shall always have $Q^i_{jk((m))} = 0$ and $Q^i_{j((m))} = 0$ and in the light of these observations ,we shall have

$$Q^s_{jk}\Pi^i_{hms} - 2Q^i_{s[k}\Pi^s_{j]hm} = 0 \quad . \tag{2.18}$$

Transvecting (2.10) by $\dot{x}^h$ and thereafter using (1.16) along with the fact that $\pounds_v \dot{x}^i = 0$,we get

$$\pounds_v Q^i_{jk}(x,\dot{x}) = 2\{\in_{h[((k))]} \delta^i_{j]}\dot{x}^h + \dot{x}^h \in_{h[j((k))]}\} \quad . \tag{2.19}$$

With the help of equations (1.16) ,(2.7) ,(2.18) and (2.19) ,we get

$$Q^s_{jk} \in_{hs} \delta^i_m + Q^i_{jk} \in_{hm} -2\{Q^i_{h[k} \in_{j]m} + Q^i_{m[j} \in_{k]h}\} - \tag{2.20}$$

$$-\{\in_{r((s))} \dot{x}^r \delta^i_{[k} - \dot{x}^i(\in_{s<((k))>} - \in_{k<((s))>} \Pi^i_{j]hm}\} = 0$$

transvecting (2.20) by $\dot{x}^m$ and thereafter using the set of equation given by (1.16) and (2.6) ,we get

$$Q^s_{jk} \in_{hs} \dot{x}^i + Q^i_{jk} \in_h -2\{Q^i_{h[k} \in_{j]} + Q^i_{[k} \in_{j]h}\} = 0 \tag{2.21}$$

in the light of (2.21) ,we can therefore state:

THEOREM (2.4):

In a Finsler space F_n, the equation (2.21) always holds provided the special projective symmetric Finsler space admits a special projective motion characterized by the infinitesimal point transformation (2.4).

3. H- CURVATURE COLLINEATION AND CONFORMAL MOTION

If the infinitesimal point transformation (2.4) implies that the magnitude of the vectors defined in the same tangent space are proportional and the angle between the two directions is the same with respect to the respective metrics then it is called a conformal motion in a Finsler space F_n .The variation of $G^i_{jk}(x,\dot{x})$ under the infinitesimal point change

(2.4) in $\pounds_v G^i_{jk}(x,\dot{x})$ and the variation under the conformal change $\bar{G}^i_{jk}(x,\dot{x})$,the two transformations will coincide if the corresponding variations are the same .

The necessary and sufficient condition in order that the infinitesimal point transformation characterized by (2.4) be a H-conformal motion in F_n is that the Lie-derivative of the Berwald's connection coefficient $G^i_{jk}(x,\dot{x})$ satisfies the relation [3]

$$\pounds_v G^i_{hk} = \delta^i_h \in_k + \delta^i_k \in_h - \in^i g_{kh} \quad , \tag{3.1}$$

where , $\in^i(x)$ is a non-zero contravariant vector field depending on positional coordinates only and satisfies the relation

$$\in^i = g^{ik} \in_k \quad . \tag{3.2}$$

In view of (3.1) the following relations can also be obtained

$$\text{(a)} \quad \pounds_v G^i_{shk} = -2 \in^i C_{shk} \tag{3.3}$$

and (b) $\pounds_v G^i_k = \dot{x}^i \in_k - \in^i g_{hk} \dot{x}^h$

where,

$$\text{(a)} \quad C_{shk} = \frac{1}{2} \dot{\partial}_s g_{hk} \quad \text{and} \quad \text{(b)} \quad \in_h = \partial_h \in \quad . \tag{3.4}$$

We shall now investigate the conditions under which a conformal motion becomes a curvature collineation .The commutation formula (1.9 c) in view of (3.1) assumes the following form

$$\begin{aligned} \pounds_v H^i_{hjk} = {} & \delta^i_h \in_{j(k)} + \delta^i_j \in_{h(k)} - \in^i_{(k)} g_{hj} - \in^i g_{hj(k)} - \\ & - \delta^i_k \in_{h(j)} + \delta^i_h \in_{k(j)} + \in^i_{(j)} g_{kh} + \in^i g_{kh(j)} - \\ & - \delta^l_k \in_b G^i_{hjl} \dot{x}^b - \delta^l_b \in_k G^i_{hjl} \dot{x}^b + \in^l g_{kb} G^i_{hjl} \dot{x}^b + \\ & + \delta^l_k \in_b G^i_{khl} \dot{x}^b + \delta^l_b \in_j G^i_{hkl} \dot{x}^b - \in^l g_{kb} G^i_{khl} \dot{x}^b \end{aligned} \tag{3.5}$$

using equations (1.8) ,(1.11) ,(2.8) and the fact that $\dot{x}^i_{(k)} = 0$,we get the following from (3.5)

$$\begin{aligned} \pounds_v H^i_{hjk} = {} & \delta^i_k \in_{h(j)} - \delta^i_j \in_{h(k)} + \in^i_{(k)} g_{hj} - \in^i_{(j)} g_{kh} + \\ & + \in^i (g_{jh(k)} - g_{kh(j)}) . \end{aligned} \tag{3.6}$$

we now introduce in (3.6) ,the formula analogous to (2.5) and get

$$\delta^i_k \in_{h(j)} - \delta^i_j \in_{h(k)} + \in^i_{(k)} g_{hj} - \in^i_{(j)} g_{kh} + \in^i (g_{jh(k)} - g_{kh(j)}) = 0 \tag{3.7}$$

allowing a contraction in (3.7) with respect to the indices i and j ,we get

$$(1-n)\in_{h(k)} ++\in^{i}_{(k)} g_{ih}-\in^{i}_{(i)} g_{kh}+\in^{i} (g_{ih(k)}-g_{kh(i)})=0 . \tag{3.8}$$

In the light of (3.8) ,we can therefore state:

THEOREM (3.1):

In an F_n ,the equation (3.8) holds provided the space under consideration admits a conformal motion characterized by (3.1) and a curvature collineation characterized by (2.5) .

Allowing a contraction in (3.6) with respect to the indices I and k and thereafter using (1.11) ,we get

$$£_v H_{hj} = (n-1)\in_{h(j)} +\in^{i}_{(j)} g_{ih}+\in^{i} (g_{jh(i)}-g_{ih(j)}) . \tag{3.9}$$

Taking into account the equation analogous to (2.6) in (3.9) ,we get

$$(n-1)\in_{h(j)} +\in^{i}_{(j)} g_{ih}+\in^{i} (g_{jh(i)}-g_{ih(j)})=0 . \tag{3.10}$$

In the light of (3.10) ,we can therefore state :

Theorem(3.2):

In a Finsler space F_n ,the equation (3.10) holds provided the space under consideration admits a conformal motion characterized by (3.1) and a Ricci collineation given by (2.6) .

We now apply the commutation formula (1.9b) to the deviation tensor field $H^i_j(x,\dot{x})$ and thereafter use the fact that $H^i_{j(m)}=0$ along with (3.1) and get

$$£_v (H^i_j)_{(k)} = H^h_j(\delta^i_k \in_h +\delta^i_h \in_k -\in^i g_{kh}) - H^i_h(\delta^h_k \in_j +\delta^h_j \in_k -\in^h g_{kj}) -$$
$$-(\dot{\partial}_h H^i_j)(\delta^h_k \in_s +\delta^h_s \in_k -\in^h g_{ks})\dot{x}^s . \tag{3.11}$$

We now make use of (1.11) in (3.11) and get

$$£_v (H^i_j)_{(k)} = H^i_k \in_j -H^i_h \in^h g_{ks} -\delta^i_k H^h_j \in_h +\in^i H_{jk} +$$
$$+(\dot{\partial}_s H^i_j \in_k -\dot{\partial}_h H^i_j g_{ks}\dot{x}^h)\dot{x}^s \quad , \tag{3.12}$$

where

$$H_{jk} = g_{kh} H^h_j \tag{3.13}$$

we now allow a contraction in (3.12) with respect to the indices i and j and get

$$(£_v H^i_i)_{(k)} = \{(\dot{\partial}_s H^i_i)\in_k -(\dot{\partial}_h H^i_i) g_{ks}\}\dot{x}^s . \tag{3.14}$$

Therefore we can state

THEOREM(3.3):

In a Finsler space F_n , (3.14) holds provided the space under consideration admits a conformal motion characterized by (3.1) .

It is almost obvious from (3.1) that for an affine motion characterized by $\pounds_v G^i_{jk}(x,\dot{x})=0$,the covariant and contravariant vectors $\in_h (x)$ and $\in^h (x)$ must vanish identically and as such we can therefore state:

COROLLARY(3.1):

In a Finsler space F_n , $(\pounds_v H^i_i)_{(k)}$ vanishes provided the space under consideration admits a conformal motion characterized by (3.1) and an affine motion characterized by $\pounds_v G^i_{jk}(x,\dot{x})=0$.

We now allow a transvection in (3.6) by $\dot{x}^h$ and thereafter use (1.11) along with the fact that $\pounds_v \dot{x}^h=0$ and get

$$\pounds_v H^i_{jk}=\{\delta^i_k \in_{h(j)} -\delta^i_j \in_{h(k)} +\in^i_{(k)} g_{jh}-\in^i_{(j)} g_{kh}+$$

$$+\in^i (g_{jh(k)}-g_{kh(j)})\}\dot{x}^h \quad . \tag{3.15}$$

We now apply (2.8) and(3.4b) in (3.15) and get

$$\pounds_v H^i_{jk}=\{\in^i_{(k)} g_{jh}-\in^i_{(j)} g_{kh}+\in^i (g_{jh(k)}-g_{kh(j)})\}\dot{x}^h \quad . \tag{3.16}$$

Applying the commutation formula (1.9) to H^i_{jk} ,we get

$$\dot{\partial}_l (H^i_{jk(m)})-(\dot{\partial}_h H^i_{jk})_{(m)}=H^s_{jk}G^i_{hms}-H^i_{sk}G^s_{jhm}+H^i_{sj}G^s_{khm} \quad . \tag{3.17}$$

Now using (1.11) in (3.17) ,we get (here we have taken into account the fact that if the Berwald's curvature tensor satisfies $H^i_{hjk(m)}=0$ then such a space is called a symmetric Finsler space and in such a space ,we also have $H^i_{jk(m)}=0$)

$$H^s_{jk}G^i_{hms}+H^i_{sj}G^s_{khm}-H^i_{sk}G^s_{jhm}=0 \quad . \tag{3.18}$$

We now apply the operation of Lie-derivation in (3.18) and then use (3.3a) along with (3.16) and get

$$G^i_{hms}\{(\in^s_{(k)} g_{jh}-\in^s_{(j)} g_{kh})+\in^s (g_{jh(k)}-g_{kh(j)})\}\dot{x}^h -$$

$$-2\in^i C_{hms}H^s_{jk}+G^s_{khm}\{\in^i_{(j)} g_{sh}-\in^i_{(s)} g_{jh})+\in^i (g_{sh(j)}-g_{jh(s)})\}\dot{x}^h -$$

$$-2\in^s C_{khm}H^i_{sj}-G^h_{jhm}\{(\in^i_{(k)} g_{sh}-\in^i_{(s)} g_{kh})+\in^i (g_{sh(k)}-g_{kh(s)})\}\dot{x}^h +$$

$$+2H^i_{sk}\in^s C_{jhm}=0. \tag{3.19}$$

Using (1.11) and (2.8) in (3.19) ,we get

$$H^s_{jk}C_{hms}\in^i +2\in^s H^i_{s[j}C_{k]hm}=0 \quad . \tag{3.20}$$

In the light of (3.20) ,we can therefore state :

THEOREM(3.4):

In a symmetric Finsler space F_n , (3.20) always holds provided such a space admits a conformal motion characterized by (3.1) .

4. Q-CONFORMAL MOTION

The necessary and sufficient condition in order that the infinitesimal point transformation characterized by (2.4) be a Q-conformal motion in a Finsler space F_n is that the Lie-derivative of the projective connection coefficient $\Pi^i_{hk}(x,\dot{x})$ should satisfy the following relation

$$\pounds_v \Pi^i_{hk} = \delta^i_k \in_h + \delta^i_h \in_k - \in^i g_{hk} \quad , \tag{4.1}$$

where $\in_i (x)$ and $\in^i (x)$ are non- zero covariant and contra variant vector fields respectively .Keeping in mind the definition given by (4.1) ,we can easily deduce the following :

$$\text{(a)} \quad \pounds_v \Pi^i_{shk} = -2 \in^i C_{shk} \tag{4.2}$$

and

$$\text{(b)} \quad \pounds_v \Pi^i_k = \dot{x}^i \in_k - \in^i g_{hk} \dot{x}^h$$

where

$$\text{(a)} \quad C_{shk} = \frac{1}{2} \dot{\partial}_s g_{hk} \quad \text{and} \quad \text{(b)} \quad \Pi^i_k = \Pi^i_{hk} \dot{x}^h \quad . \tag{4.3}$$

We shall now investigate the conditions under which a Q-conformal motion becomes a Q-curvature collineation .The commutation formula (2.3c) in view of (4.1) reduces into the following form

$$\begin{aligned}\pounds_v Q^i_{hjk} = {} & \delta^i_h \in_{j((k))} + \delta^i_j \in_{h((k))} - \in^i_{((k))} g_{hj} - \in^i g_{hj((k))} - \delta^i_k \in_{h((j))} - \\ & -\delta^i_h \in_{k((j))} + \in^i_{((j))} g_{kh} + \in^i g_{hj((k))} + \in^l g_{kb} \Pi^i_{hjl} \dot{x}^b + \\ & + \delta^l_k \in_b \Pi^i_{hkl} \dot{x}^b + \delta^l_b \in_b \Pi^i_{hjl} \dot{x}^b - \in^l g_{kb} \dot{x}^b \Pi^i_{jhl} \quad .\end{aligned} \tag{4.4}$$

Using (1.6) and (2.8) in (4.4) ,we get

$$\begin{aligned}\pounds_v Q^i_{hjk} = {} & \delta^i_k \in_{h((j))} - \delta^i_j \in_{h((k))} + \in^i_{((k))} g_{jh} - \in^i_{((j))} g_{kh} + \\ & + \in^i (g_{jh((k))} - g_{kh((j))}) \quad .\end{aligned} \tag{4.5}$$

Using (2.5) in (4.5) ,we get

$$\delta^i_k \in_{h((j))} - \delta^i_j \in_{h((k))} + \in^i_{((k))} g_{jh} - \in^i_{((j))} g_{kh} + \in^i (g_{jh((k))} - g_{kh((j))}) = 0 \quad . \tag{4.6}$$

We now contract (4.6) with respect to the indices i and j and get

$$(1-n) \in_{h((k))} + 2 \in^i_{[((k))} g_{i]h} + 2 \in^i g_{h[i((k))]} = 0 \quad . \tag{4.7}$$

We now transvect (4.7) by $\dot{x}^h$ and thereafter use (2.8) and the fact that $\dot{x}^i_{((k))} = 0$,we get

$$\in^i_{[((k))} g_{i]h} + \in^i g_{h[i((k))]} = 0 \tag{4.8}$$

in the light of (4.8) ,we can therefore state:

THEOREM(4.1):

In a Finsler space F_n , (4.8) always holds provided the space under consideration admits a Q-conformal motion characterized by (4.1) and a Q-curvature collineation characterized by (2,5).

We now allow a contraction in (4.5) with respect to the indices i and k and thereafter use (1.16) and get

$$£_v Q_{hj} = (n-1) \in_{h((j))} + 2 \in^i_{[((k))} g_{j]h} + 2 \in^i g_{h[j((i))]} \;. \tag{4.9}$$

Using (2.6) in (4.9) ,we get

$$(n-1) \in_{h((j))} + 2 \in^i_{[((k))} g_{j]h} + 2 \in^i g_{h[j((i))]} = 0 \;. \tag{4.10}$$

As per provisions of (4.10) ,we can therefore state:

THEOREM(4.2):

In a Finsler space F_n , (4.10) always holds provided the space under consideration admits a Q-conformal motion characterized by (4.1) and a Q- Ricci collineation characterized by (2.6).

We now apply the commutation formula (2.3b) to the projective entity $Q^i_j(x, \dot{x})$ and thereafter use (2.8) and (4.1) and get

$$\begin{aligned}(£_v Q^i_{j((k))}) = Q^i_k \in_j - Q^i_h \in^h g_{kj} - \delta^i_k Q^h_j \in_h + \in^i Q_{jk} + \\ + \dot{x}^s (\dot{\partial}_s Q^i_j \in_k - \partial_j Q^i_h g_{ks} \dot{x}^h)\end{aligned} \tag{4.11}$$

where

$$Q_{jh} = g_{kh} Q^k_j \quad . \tag{4.12}$$

We now allow a contraction in (4.11) with respect to the indices i and j and get

$$(£_v Q^i_i)_{((i))} = \{(\dot{\partial}_s Q^i_i) \in_k - (\dot{\partial}_h Q^i_i) g_{ks} \dot{x}^h\} \dot{x}^s \quad . \tag{4.13}$$

In the light of (4.13) ,we can therefore state.

THEOREM(4.3):

In a Finsler space (4.13) always holds provided the space under consideration admits a conformal motion characterized by (4.1).

CONCLUSION

This presentation has been divided into four sections of which the first section is introductory ,in the second section we have discussed projective (Q) curvature collineation and have derived results telling as to what will happen when the special projective motion becomes a projective curvature collineation and also becomes a projective Ricci curvature collineation and the sequel have derived certain more results telling what relationships will hold when a projective Finsler space admits a special projective motion .The third section of the communication has been devoted to the study of H-curvature collineation and H-conformal motion in a Finsler space .In This section ,we have derived results telling as to what relationships will hold when the space under consideration admits a H-conformal motion and H-curvature collineation and in the sequel have also derived the relationships which hold when the space admits both conformal motion and affine motion .The fourth section has been devoted to the study of Q-(projective) conformal motion .In this section too we have derived results which hold when the space admits Q- conformal motion and Q- curvature collineation .

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