The Influence of Nonlinear Damping and Geometric Imperfections on Oscillatory Systems

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Abstract: The behaviour of systems that oscillate is an important area of study in so many fields due to its vast applications. It has a wide range of applications that cross across man-made and natural systems. There are many factors that can affect oscillatory systems. In this research, two factors are considered: nonlinear damping and geometric imperfections (the deviation and incompleteness of a system or components). Imperfections could be from the shape, the length and other irregularities which might be caused in the production process. Oscillatory systems are seen in physics, electrical engineering, mechanical engineering, chemical engineering, and biology. The forced free Van der Pol oscillator problem that models many oscillatory systems has been used to illustrate the effect of these irregularities generally known as imperfections, and nonlinear damping in oscillatory systems. The Van der Pol oscillator problem is solved using two-timing regular parameter perturbation method. It is observed that geometric imperfections increase the amplitude of oscillation of the system thereby increasing the period of oscillation and damping alters the amplitude of the oscillatory system decreasing it as it increases.

Keywords: Oscillatory systems, geometric imperfections, amplitude, period, irregularities, regular parameter perturbation, nonlinear damping.

1. Introduction

Van der Pol oscillator in the investigation of the behavior of dynamical system is often referred as a non-conservative and oscillating system with nonlinear damping. Balthasar Van der Pol, a Dutch Electrical Engineer and Physicist while working at Philips, proposed Van der Pol oscillator (Cartwright [5]). It models a wide range of dynamic characteristics observed in physical, chemical and biology systems. In biology, Fitzhugh [14] and Nagumo et al [26] extended the equation in a plana field as a model for action potentials of neurons. Buonomo [4] presented an analytic continuation method for finding the periodic solutions of Van der pol equation and concluded that the method is very efficient to find the amplitude and period at all powers of the parameter ε . Panayotounakos et al [29], in their research on the lack of analytic solution of the Van der pol oscillator, their result showed that Van der pol does not have an exact analytic solution. Van der Pol problem has been solved using different solution methods. Davis and Alfried [8] used a perturbation method first developed by Cochran and extended Nayfeh to obtain a uniformly valid second approximation for arbitray initial conditions when the coefficient ε is small. Mohammadi et al [25], converted the nonlinear Duffing-Van der Pol equation to another nonlinear volterra integral equation of the second kind using integration and approximate with the hybrid Legendre polynomials and block-pulse function. John et al [19] considered the slow flow and bifurcation of forced Van der Pol. Sergey [30] and Matthew et al [23] used Van der Pol equation in their respective research.

Khan [20] applied homotopy perturbation method to solve Van der Pol nonlinear differential equation with different boundary conditions. Bachir and George [3] in their research, combined homotopy perturbation method and natural decomposition method to obtain a numerical solution of Van der Pol oscillator problem. Maha et al [22] studied the general form of stochastic Van der Pol equation (SVDP) under an external excitation described by white noise using Wiener-chaos expansion technique and Wiener-Hermite expansion technique. Cveticanin [6] investigated on the truly nonlinear oscillator with positive and negative damping and he applied harmonic balance method. Hiroshi et al [17] analyzed Van der Pol system with columb friction using the method of multiple scales. The behaviour of a hybrid Rayleigh Van der Pol-Duffing oscillator with a PD controller was studied by Amen et al [1]. Lucero and Schoentgen [21] researched on modeling vocal fold asymmetrics with coupled Van der Pol oscillators. The investigation of the stability conditions of the cubic damping Van der Pol-Duffing oscillator using the homotopy perturbation method with the frequency-expansion technology was done by El-Dib [10]. Elfouly and Sohaly [11] applied taylor series to Van der Pol model as a two-delay differential equation to study the LRC circuit.

Geometrical imperfections in systems have attracted interest of many researchers. Ghadimi et al [15] investigated the thermal flutter characteristics of an imperfect cantilever plate under aerodynamic load and in their research, they modeled geometric imperfections as strain energy that causes a reduction in the structural stiffness. Effect of imperfections and damping on the type of nonlinearity of circular plates and shallow spherical shells was studied by Cyril et al [7]. The developments of the effect of geometric imperfections on the amplitude and the phase angle of the nonlinear vibrations of thin rectangular plates parametrically excited was seen Mihai et al [24]. Geometric imperfections also affect buckling of structures; Ette [12,13], Jacek [18], Onuoha and Ette [27], Onuoha [28]. Hamed and Mergen [16] studied the nonlinear dynamical behaviour of geometrically imperfect microplates based on modified couple stress theory. Other investigations on the effects of geometric imperfections were carried out by Ellcol [9] on the vibrations of anisotropic cylindrical shells and Aymen et al [2] on the nonlinear static and dynamical behaviour of capacitive micromechanical ultrasonic transducers.

2. Forced free Van der Pol Problem

$$\frac{d^2x}{dt^2}+\delta\left(1-x^2\right)\frac{dx}{dt}+x=0 \tag{1}$$

$$x(0)=0$$
$$\frac{dx(0)}{dt}=\varepsilon \tag{2}$$

In this research, the dependent variable, $x(t)$ which is the spatial variable represents the amplitude of the oscillatory system while the independent variable, t is the time variable. Two timing scale and regular parameter perturbation will be applied to equation (1) and (2)

3. Two timing scale and regular parameter perturbation

Introducing the timing scale, we let

$$\tau(t)=\delta t \tag{3}$$

$$x(t)=z(t,\tau) \tag{4}$$

For regular parameter perturbation, we let

$$z(t,\tau)=\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij}\varepsilon^i\delta^j \tag{5}$$

Using equation (3), equation (4) and equation (5), equation (1) and (2) respectively becomes

$$\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,tt}\varepsilon^i\delta^j+2\delta\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,t\tau}\varepsilon^i\delta^j+\delta^2\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,\tau\tau}\varepsilon^i\delta^j+\delta\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,t}\varepsilon^i\delta^j+\delta^2\sum_{\substack{i=1\\j-0}}^{\infty} z_{ij,\tau}\varepsilon^i\delta^j-$$
$$\delta\left(\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij}\varepsilon^i\delta^j\right)^2\left(\delta\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,t}\varepsilon^i\delta^j+\delta^2\sum_{\substack{i=1\\j-0}}^{\infty} z_{ij,\tau}\varepsilon^i\delta^j\right)+\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij}\varepsilon^i\delta^j=0 \tag{6a}$$

$$\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij}(0,0)\varepsilon^i\delta^j=0$$
$$\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,t}(0,0)\varepsilon^i\delta^j+\delta\sum_{\substack{i=1\\j=0}}^{\infty} z_{ij,\tau}(0,0)\varepsilon^i\delta^j=\varepsilon \tag{6b}$$

Solving equation (6a) with equation (6b), we equate coefficients of $\varepsilon^i \delta^j \quad (i=1,2,3;\ j=0,1,2)$ and get

$$\left(\varepsilon^1\delta^0\right): z_{10,tt} + z_{10} = 0 \qquad z_{10}(0,0)=0 \qquad z_{10,t}(0,0)=1 \tag{7}$$

$$\left(\varepsilon^1\delta^1\right): z_{11,tt} + 2z_{10,t\tau} + z_{10,t} + z_{11} = 0 \qquad z_{11}(0,0)=0 \qquad z_{11,t}(0,0)+z_{10,\tau}(0,0)=0 \tag{8}$$

$$\left(\varepsilon^1\delta^2\right): z_{12,tt} + 2z_{11,t\tau} + z_{10,\tau\tau} + z_{11,t} + z_{10,\tau} + z_{12} = 0 \qquad z_{12}(0,0)=0 \qquad z_{12,t}(0,0)+z_{11,\tau}(0,0)=0 \tag{9}$$

$$\left(\varepsilon^2\delta^0\right): z_{20,tt} + z_{20} = 0 \qquad z_{20}(0,0)=0 \qquad z_{20,t}(0,0)=0 \tag{10}$$

$$\left(\varepsilon^2\delta^1\right): z_{21,tt} + 2z_{20,t\tau} + z_{20,t} + z_{21} = 0 \qquad z_{21}(0,0)=0 \qquad z_{21,t}(0,0)+z_{20,\tau}(0,0)=0 \tag{11}$$

$$\left(\varepsilon^2\delta^2\right): z_{22,tt} + 2z_{21,t\tau} + z_{20,\tau\tau} + z_{21,t} + z_{20,\tau} + z_{22} = 0 \qquad z_{22}(0,0)=0 \qquad z_{22,t}(0,0)+z_{21,\tau}(0,0)=0 \tag{12}$$

$$\left(\varepsilon^3\delta^0\right): z_{30,tt} + z_{30} = 0 \qquad z_{30}(0,0)=0 \qquad z_{30,t}(0,0)=0 \tag{13}$$

$$\left(\varepsilon^3\delta^1\right): z_{31,tt} + 2z_{30,t\tau} + z_{30,t} - \left(z_{10}\right)^2 z_{10,t} + z_{31} = 0 \qquad z_{31}(0,0)=0 \qquad z_{31,t}(0,0)+z_{30,\tau}(0,0)=0 \tag{14}$$

$$\left(\varepsilon^3\delta^2\right): z_{32,tt} + 2z_{31,t\tau} + z_{30,\tau\tau} + z_{31,t} + z_{30,\tau} - \left(z_{10}\right)^2 z_{11,t} - \left(z_{10}\right)^2 z_{10,\tau} + z_{32} = 0 \qquad z_{32}(0,0)=0 \qquad z_{32,t}(0,0)+z_{31,\tau}(0,0)=0 \tag{15}$$

Solution to equation (7)

Solving equation (7), we get

$$z_{10}(t,\tau) = A_{10}(\tau)\cos t + B_{10}(\tau)\sin t \tag{16a}$$

$$A_{10}(0) = 0 \tag{16b}$$

$$B_{10}(0)=1 \tag{16c}$$

Solution to equation (8)

$$z_{11,tt}+z_{11}=-2\left(-A_{10}'(\tau)\sin t+B_{10}'(\tau)\cos t\right)-\left(-A_{10}(\tau)\sin t+B_{10}(\tau)\cos t\right) \tag{17}$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A_{10}'(\tau)+\frac{1}{2}A_{10}(\tau)=0 \tag{18}$$

Next, we solve equation (18) to get

$$A_{10}(\tau)=k_1e^{-\frac{1}{2}\tau} \tag{19a}$$

Applying equation (16b) on equation (19a), we get

$$A_{10}(\tau)=0 \tag{19b}$$

For $\cos t$

$$B_{10}'(\tau)+\frac{1}{2}B_{10}(\tau)=0 \tag{20}$$

Next, we solve equation (20) to get

$$B_{10}(\tau)=k_2e^{-\frac{1}{2}\tau} \tag{21a}$$

Applying equation (16c) on equation (21a), we get

$$B_{10}(\tau)=e^{-\frac{1}{2}\tau} \tag{21b}$$

The remaining part of equation (17) is

$$z_{11,tt}+z_{11}=0 \tag{22}$$

Solving equation (22), we get

$$z_{11}(t,\tau)=A_{11}(\tau)\cos t+B_{11}(\tau)\sin t \tag{23a}$$

$$A_{11}(0)=0 \tag{23b}$$

$$B_{11}(0)=0 \tag{23c}$$

Solution to equation (9)

$$\begin{aligned} z_{12,tt}+z_{12}=&-2\left(-A_{11}'(\tau)\sin t+B_{11}'(\tau)\cos t\right)-B_{10}''(\tau)\sin t- \\ &\left(-A_{11}(\tau)\sin t+B_{11}(\tau)\cos t\right)-B_{10}'\sin t \end{aligned} \tag{24}$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A_{11}'(\tau)+\frac{1}{2}A_{11}(\tau)=\frac{1}{2}\left(B_{10}''+B_{10}'\right) \quad (25a)$$

Next, we solve equation (25a) to get

$$A_{11}(\tau)=0 \quad (25b)$$

For $\cos t$

$$B_{11}'(\tau)+\frac{1}{2}B_{11}(\tau)=0 \quad (26)$$

Next, we solve equation (26) to get

$$B_{11}(\tau)=k_4 e^{-\frac{1}{2}\tau} \quad (27a)$$

Applying equation (23c) on equation (27a), we get

$$B_{11}(\tau)=0 \quad (27b)$$

The remaining part of equation (24) is

$$z_{12,tt}+z_{12}=0 \quad (28)$$

Solving equation (28), we get

$$z_{12}(t,\tau)=A_{12}(\tau)\cos t+B_{12}(\tau)\sin t \quad (29a)$$

$$A_{12}(0)=0 \quad (29b)$$

$$B_{12}(0)=0 \quad (29c)$$

Solution to equation (10)

Solving equation (10), we get

$$z_{20}(t,\tau)=A_{20}(\tau)\cos t+B_{20}(\tau)\sin t \quad (30a)$$

$$A_{20}(0)=0 \quad (30b)$$

$$B_{20}(0)=0 \quad (30c)$$

Solution to equation (11)

$$z_{21,tt}+z_{21}=-2\left(-A_{20}'(\tau)\sin t+B_{20}'(\tau)\cos t\right)-\left(-A_{20}(\tau)\sin t+B_{20}(\tau)\cos t\right) \quad (31)$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A_{20}'(\tau)+\frac{1}{2}A_{20}(\tau)=0 \quad (32)$$

Next, we solve equation (32) to get

$$A_{20}(\tau)=k_5 e^{-\frac{1}{2}\tau} \quad (33a)$$

Applying equation (30b) on equation (33a), we get

$$A_{20}(\tau) = 0 \tag{33b}$$

For $\cos t$

$$B'_{20}(\tau) + \frac{1}{2}B_{20}(\tau) = 0 \tag{34}$$

Next, we solve equation (34) to get

$$B_{20}(\tau) = k_6 e^{-\frac{1}{2}\tau} \tag{35a}$$

Applying equation (30c) on equation (35a), we get

$$B_{20}(\tau) = 0 \tag{35b}$$

The remaining part of equation (31) is

$$z_{21,tt} + z_{21} = 0 \tag{36}$$

Solving equation (36), and applying the conditions we get

$$z_{21}(t,\tau) = A_{21}(\tau)\cos t + B_{21}(\tau)\sin t \tag{37a}$$

$$A_{21}(0) = 0 \tag{37b}$$

$$B_{21}(0) = 0 \tag{37c}$$

Solution to equation (12)

$$\begin{aligned} z_{22,tt} + z_{22} &= -2\left(-A'_{21}(\tau)\sin t + B'_{21}(\tau)\cos t\right) - \left(A''_{20}(\tau)\cos t + B''_{20}(\tau)\sin t\right) \\ &\left(-A_{21}(\tau)\sin t + B_{21}(\tau)\cos t\right) - \left(A'_{20}(\tau)\cos t + B'_{20}(\tau)\sin t\right) \end{aligned} \tag{38}$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A'_{21}(\tau) + \frac{1}{2}A_{21}(\tau) = 0 \tag{39}$$

Next, we solve equation (39) to get

$$A_{21}(\tau) = k_7 e^{-\frac{1}{2}\tau} \tag{40a}$$

Applying equation (37b) on equation (40a), we get

$$A_{21}(\tau) = 0 \tag{40b}$$

For $\cos t$

$$B'_{21}(\tau) + \frac{1}{2}B_{21}(\tau) = 0 \tag{41}$$

Next, we solve equation (41) to get

$$B_{21}(\tau) = k_8 e^{-\frac{1}{2}\tau} \qquad (42a)$$

Applying equation (37c) on equation (42a), we get

$$B_{21}(\tau) = 0 \qquad (42b)$$

The remaining part of equation (38) is

$$z_{22,tt} + z_{22} = 0 \qquad (43)$$

Solving equation (43) and applying the conditions, we get

$$z_{22}(t,\tau) = A_{22}(\tau)\cos t + B_{22}(\tau)\sin t \qquad (44a)$$

$$A_{22}(0) = 0 \qquad (44b)$$

$$B_{22}(0) = 0 \qquad (44c)$$

Solution to equation (13)

Solving equation (13), we get

$$z_{30}(t,\tau) = A_{30}(\tau)\cos t + B_{30}(\tau)\sin t \qquad (45a)$$

Applying the conditions, we get

$$A_{30}(0) = 0 \qquad (45b)$$

$$B_{30}(0) = 0 \qquad (45c)$$

Solution to equation (14)

$$\begin{aligned} z_{31,tt} + z_{31} = {} & -2\left(-A'_{30}(\tau)\sin t + B'_{30}(\tau)\cos t\right) - \left(-A_{30}(\tau)\sin t + B_{30}(\tau)\cos t\right) - \\ & \frac{1}{2}(B_{10})^3 \cos t + \frac{1}{4}(B_{10})^3 \cos 3t \end{aligned} \qquad (46)$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A'_{30}(\tau) + \frac{1}{2}A_{30}(\tau) = 0 \qquad (47)$$

Next, we solve equation (47) to get

$$A_{30}(\tau) = k_9 e^{-\frac{1}{2}\tau} \qquad (47a)$$

Applying equation (45b) on equation (47a), we get

$$A_{30}(\tau) = 0 \qquad (47b)$$

For $\cos t$

$$B'_{30}(\tau) + \frac{1}{2}B_{30}(\tau) = \frac{1}{2}(B_{10})^3 \qquad (48)$$

Next, we solve equation (48) to get

$$B_{30}(\tau) = -\frac{1}{2}e^{-\frac{3}{2}\tau} \tag{49}$$

The remaining part of equation (46) is

$$z_{31,tt} + z_{31} = \frac{1}{4}(B_{10})^3 \cos 3t \tag{50}$$

Solving equation (50), and applying the conditions we get

$$z_{31}(t,\tau) = A_{31}(\tau)\cos t + B_{31}(\tau)\sin t - \frac{1}{8}G_1(\tau)\cos 3t \tag{51a}$$

where

$$G_1(\tau) = \frac{1}{4}(B_{10})^3$$

$$A_{31}(0) = \frac{1}{8}G_1(0) = \frac{1}{32} \tag{51b}$$

$$B_{31}(0) = 0 \tag{51c}$$

Solution to equation (15)

$$\begin{aligned} z_{32,tt} + z_{32} = &-2\left(-A'_{31}(\tau)\sin t + B'_{31}(\tau)\cos t + \frac{3}{8}G'_1(\tau)\sin 3t\right) - \left(A''_{30}(\tau)\cos t + B''_{30}(\tau)\sin t\right) - \\ &\left(-A_{31}(\tau)\sin t + B_{31}(\tau)\cos t + \frac{3}{8}G_1(\tau)\sin 3t\right) - \left(A'_{30}(\tau)\cos t + B'_{30}(\tau)\sin t\right) - \\ &(B_{10})^2 A_{11}\sin t + \frac{1}{2}(B_{10})^2 A_{11}(\sin 3t + \sin t) + (B_{10})^2 B_{11}\cos t + (B_{10})^2 B'_{10}\sin t - \\ &\frac{1}{2}(B_{10})^2 B_{11}(\cos 3t + \cos t) - \frac{1}{2}(B_{10})^2 B'_{10}(\sin 3t + \sin t) \end{aligned} \tag{52}$$

To obtain a bounded solution in t, we equate to zero the coefficients of $\sin t$ and $\cos t$

For $\sin t$

$$A'_{31}(\tau) + \frac{1}{2}A_{31}(\tau) = \frac{1}{2}\left\{B''_{30} + B'_{30} + \frac{1}{2}(B_{10})^2 A_{11} - \frac{1}{2}(B_{10})^2 B'_{10}\right\} \tag{53}$$

Next, we solve equation (53) to get

$$\begin{aligned} A_{31}(\tau) &= e^{-\frac{1}{2}\tau}\int_0^\tau G_2(\tau)e^{\frac{1}{2}\tau}d\tau \\ &= \frac{13}{8}e^{-\frac{3}{2}\tau} \end{aligned} \tag{54a}$$

where

$$G_2(\tau) = \frac{1}{2}\left\{B''_{30} + B'_{30} + \frac{1}{2}(B_{10})^2 A_{11} - \frac{1}{2}(B_{10})^2 B'_{10}\right\} \tag{54b}$$

For $\cos t$

$$B'_{31}(\tau)+\frac{1}{2}B_{31}(\tau)=\frac{1}{2}\left\{A''_{30}+A'_{30}-\frac{1}{2}(B_{10})^2 B_{11}\right\}=0 \quad (55)$$

Next, we solve equation (55) to get

$$B_{31}(\tau)=k_9 e^{-\frac{1}{2}\tau} \quad (56a)$$

Applying equation (51c) on equation (56a), we get

$$B_{31}(\tau)=0 \quad (56b)$$

The remaining part of equation (52) is

$$z_{32,tt}+z_{32}=G_4(\tau)\sin 3t+G_5(\tau)\cos 3t \quad (57)$$

where

$$\begin{aligned}G_4(\tau)&=-\frac{3}{4}G'_1(\tau)-\frac{3}{8}G_1(\tau)-\frac{1}{2}(B_{10})^2 B'_{10}\\ G_5(\tau)&=-\frac{1}{2}(B_{10})^2 B_{11}\end{aligned} \quad (58)$$

Solving equation (57) and applying the conditions, we get

$$z_{32}(t,\tau)=A_{32}(\tau)\cos t+B_{32}(\tau)\sin t-\frac{1}{8}G_4(\tau)\sin 3t-\frac{1}{8}G_5(\tau)\cos 3t \quad (59a)$$

$$A_{32}(0)=\frac{1}{8}G_5(0) \quad (59b)$$

$$B_{32}(0)=\frac{3}{8}G_4(0)+A_{31}(0)-\frac{1}{8}G_1(0) \quad (59c)$$

The solution of equation (6) becomes

$$z(t,\tau)=\varepsilon\left(z_{10}+\delta z_{11}+\delta^2 z_{12}\right)+\varepsilon^2\left(z_{20}+\delta z_{21}+\delta^2 z_{22}\right)+\varepsilon^3\left(z_{30}+\delta z_{31}+\delta^2 z_{32}\right) \quad (60)$$

Substituting for $z_{ij}(t,\tau)$, $(i=1,2,3;\ j=0,1,2)$, we get

$$z(t,\tau)=\varepsilon e^{-\frac{1}{2}\delta t}\sin t-\frac{1}{2}\varepsilon^3 e^{-\frac{3}{2}\delta t}\sin t+\frac{13}{8}\varepsilon^3\delta e^{-\frac{3}{2}\delta t}\cos t-\varepsilon^3\delta\frac{1}{32}e^{-\frac{3}{2}\delta t}\cos 3t \quad (61)$$

4. Analysis

Equation (61) is the solution to equation (6a) which is obtained numerically at various values of ε, t and fixed value of δ. From equation (61), as t tends to infinity, the displacement $z(t,\tau)$ which is the amplitude has values. Graph of $z(t,\tau)$ against t at various values ε and fixed value of δ shall be plotted to know the effect of the imperfection parameter on the amplitude of an oscillatory motion.

Table 1: Computed values of the amplitude $z(t,\tau)$, at various values of time t, geometric imperfection ε and fixed value of $\delta, (\delta=0.01)$

	$z(t,\tau)$

t	$\varepsilon = 0.01$	$\varepsilon = 0.02$	$\varepsilon = 0.03$	$\varepsilon = 0.04$	$\varepsilon = 0.05$
0	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0016517	0.0033031	0.0049537	0.0066032	0.0082513
20	0.0030946	0.0061885	0.0092811	0.0123715	0.0154592
30	0.0043034	0.0096059	0.0129066	0.0172045	0.0214989
40	0.0052625	0.0115240	0.0157835	0.0210399	0.0262923
50	0.0059658	0.0119301	0.0178931	0.0238526	0.0298078
60	0.0064155	0.0128300	0.0192424	0.0256516	0.0320568
70	0.0066217	0.0132425	0.0198613	0.0264772	0.0330892
80	0.0066012	0.0132015	0.0198001	0.0263961	0.0329884
90	0.0063762	0.0127515	0.0191253	0.0254968	0.0318652
100	0.0059731	0.0119544	0.0179165	0.0238856	0.0298520
110	0.0054215	0.0108424	0.0162622	0.0216804	0.0270964
120	0.0047527	0.0095051	0.0142566	0.0190007	0.0237551
130	0.0039990	0.0079978	0.0119958	0.0159928	0.0199885
140	0.0031919	0.0063837	0.0095749	0.0127653	0.0159548
150	0.0023618	0.0047234	0.0070847	0.0094455	0.0118057
160	0.0015368	0.0030735	0.0046099	0.0061461	0.0076819

Table 1: Computed values of the amplitude $z(t,\tau)$, at various values of time t, damping δ and fixed value of the geometric imperfection $\varepsilon, (\varepsilon = 0.01)$

t	$z(t,\tau)$					
	$\delta = 0$	$\delta = 0.01$	$\delta = 0.02$	$\delta = 0.03$	$\delta = 0.04$	$\delta = 0.05$
0	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0017364	0.0016517	0.0015712	0.0014946	0.0014217	0.0013524
20	0.0034200	0.0030946	0.0028001	0.0025337	0.0022926	0.0020744
30	0.0049998	0.0043034	0.0037040	0.0031880	0.0027440	0.0023618
40	0.0064276	0.0052625	0.0043086	0.0035276	0.0028882	0.0023647
50	0.0076600	0.0059658	0.0046462	0.0036185	0.0028181	0.0021947
60	0.0086598	0.0064155	0.0047528	0.0035210	0.0026084	0.0019324
70	0.0093965	0.0066217	0.0046663	0.0032883	0.0023172	0.0016329
80	0.0098476	0.0066012	0.0044250	0.0029662	0.0019883	0.0013328
90	0.0099995	0.0063762	0.0040657	0.0025924	0.0016530	0.0010540
100	0.0098476	0.0059731	0.0036229	0.0021974	0.0013328	0.0008084
110	0.0093965	0.0054215	0.0031279	0.0018047	0.0010412	0.0006007
120	0.0086598	0.0047527	0.0026084	0.0013315	0.0007856	0.0004312
130	0.0076601	0.0039990	0.0020877	0.0010899	0.0005690	0.0002970
140	0.0064276	0.0031919	0.0015850	0.0007871	0.0003909	0.0001941
150	0.0049998	0.0023618	0.0011157	0.0005270	0.0002489	0.0001176
160	0.0034200	0.0015368	0.0006901	0.0003103	0.0001394	0.0000626

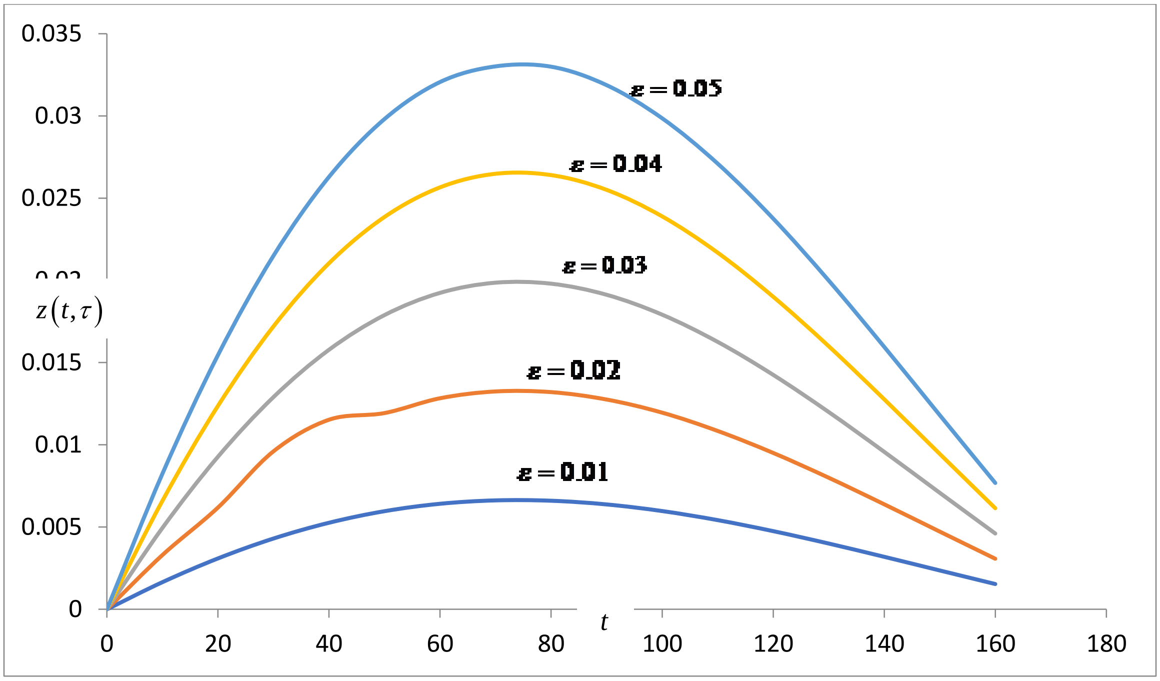

Fig 1: Variation of the amplitude, $z(t,\tau)$ with time, t at various values of geometric imperfection ε, and fixed value of damping $\delta,(\delta = 0.01)$

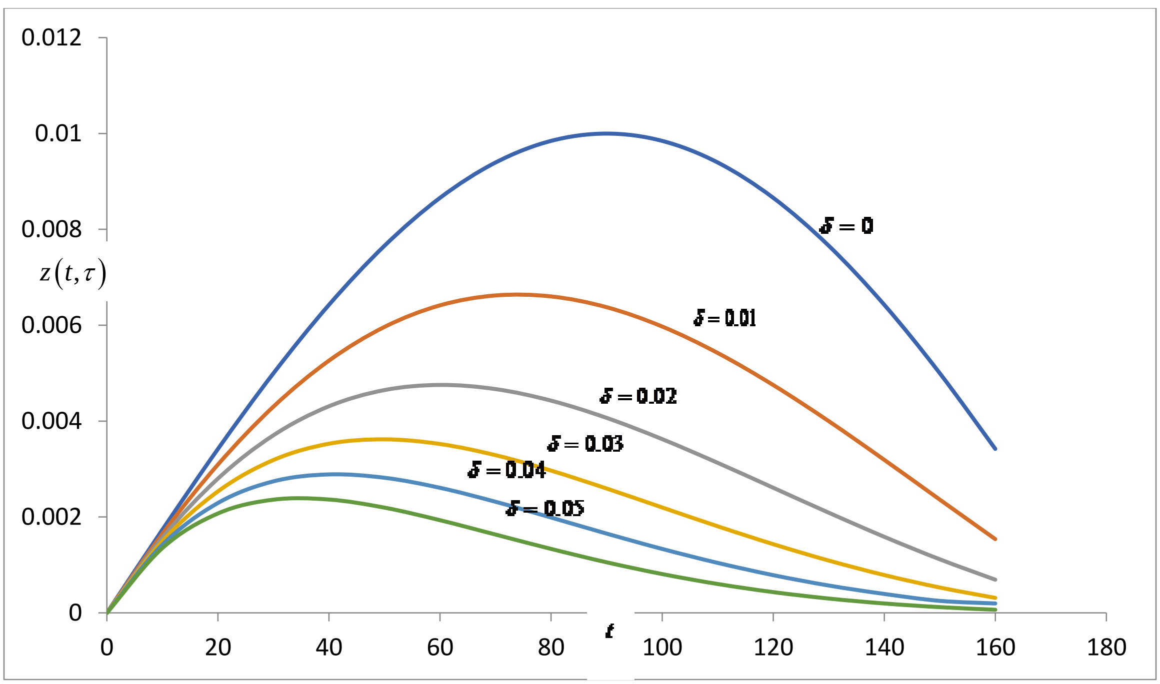

Fig 1: Variation of the amplitude, $z(t,\tau)$ with time, t at various values of damping δ, and fixed value of imperfection $\varepsilon,(\varepsilon = 0.01)$

5. Conclusion

The behavior of an oscillatory system has been investigated theoretically. Forced free Van der Pol problem was solved using regular parameter perturbation method and asymptotic expansions. The influence of damping and imperfections in the Van der Pol oscillator was considered. The efficacy of the methods, regular parameter perturbation and asymptotic expansions was seen as results were obtained using the methods. Regular parameter perturbation is an efficient mathematical tool for qualitative study of nonlinear differential equations. From the results obtained, it showed that geometric imperfections, irregularities either in shape or length which could be manufacturing fault affect the maximum displacement of the oscillatory system. Imperfection increases the maximum displacement and thereby causing the system more time to complete one cycle. Geometric imperfections presence in any system that oscillates increase the amplitude of the system making it to be easily unstable. Table I and Figure I clearly showed the effect of geometric imperfections in the oscillating system. Table II showed that damping reduces the maximum displacement, amplitude, of the oscillating system as illustrated in Figure II. Hence, it strengthens the stability of the system.

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